Distributed Energy Resources Topology Identification via Graphical Modeling

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***Abstract*—Distributed energy resources (DERs) such as photovoltaic (PV), wind, and gas generators are connected to the grid more than ever before, which introduces tremendous changes in the distribution grid. Due to these changes, it is important to understand where these DERs are connected in order to sustainably operate the distribution grid. But the exact distribution system topology is difficult to obtain due to frequent distribution grid reconfigurations and insufficient knowledge about new components. In this paper, we propose a methodology that utilizes new data from sensor-equipped DER devices to obtain the distribution grid topology. Specifically, a graphical model is presented to describe the probabilistic relationship among different voltage measurements. With power flow analysis, a mutual information-based identification algorithm is proposed to deal with tree and partially meshed networks. Simulation results show highly accurate connectivity identification in IEEE standard distribution test systems and Electric Power Research Institute (EPRI) test systems.**

Keywords*—Distributed energy resources, distribution grids, topology identification, graphical model, mutual information

I. Introduction

The electric industry is undergoing structural changes as distributed energy resources (DERs) are integrated into the distribution grid. DERs are small power sources such as photovoltaic (PV) and wind generators (renewable generation), energy storage devices (consumption flexibility), and electric vehicles (vehicle-to-grid services). As they have the potential to offer end-consumers more choices, cleaner power, and more control over energy bills, the deployment of DER is gaining momentum on a worldwide scale [1]. For example, SunShot Vision Study estimates that solar electric system will scale rapidly and potentially generate up to 14% of the nation's total electricity demand by 2030 and 27% by 2050 [2], [3].

While adding new capabilities, the DER proliferation raises great concern about the resilience of power grid. For example, power that used to flow in one direction on the distribution system - from central power plants to customers - is now also flowing back from customers to the distribution grid. Dynamic variation of voltage profiles, voltage stability, islanding, line work hazards, and distribution system operating at stability boundaries are also troubling the distribution grid operation [4]. Such problems are forcing power system engineers to rethink the architecture of power grid and transition from the traditional top-down approach to a bottom-up design since most changes happen at the customer level [5]. As a result, the concept of distribution automation is created to provide intelligent control over distribution grid level and customer level for sustainable grid operation [6].

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To achieve distribution automation, reliable and continuous monitoring of DERs is needed as it allows operators to know where the reverse power flow may happen. For example, when strategically located, DERs could potentially defer or substitute for conventional infrastructure [1]. So, understanding existing connectivity serves as a key for calculating locational benefit analysis and local grid capacity analysis for future planning purpose [1]. Further, contingency analysis needs DERs connectivity, as they help answer "what if" questions, e.g., what if solar capacity is increased by 20% [7]? Finally, rooftop solar panel may change the revenue of utilities, so some utilities want to know where they are [8].

A major challenge of DER monitoring is that the topology of the distribution grid is difficult to obtain. For example, many substations rely on a physical topology map, which may be outdated. This is because, unlike the transmission power grid [9], [10], a distribution grid can have frequent topology changes [11]–[14]. Such a topology change can be once a season [15] or once 4 weeks for MV grids [16]. If one needs to coordinate PV, the frequency can be once per 8 hours [17]. Some known changes are results of routine reconfiguration, e.g., deliberately and dynamically change of the network for best radial topology from the potentially meshed distribution topology in city networks [18], [19]. Many other changes caused by outages or manual maintenance may be unknown. For example, a field engineer may not report immediately about topology changes after repairing part of a network [20]. Additionally, even if a topology change is known or detected in the local area, the exact change may be hard to obtain [21]. Finally, although topology sensors are being used, they are placed only at special locations due to budget constraints [22].

Furthermore, there will be massive ad-hoc connections of plug-and-play DER components. For some well-maintained feeders, identifying the connectivity of DERs can be best obtained based on switch or breaker information. However, for many feeders, switch and breaker information near these DERs may not be available. This is because many of the DERs, e.g., distributed generator, do not belong to the utility. In such a case, while the utilities may not own the DER devices, they can try to obtain the sensor data from the manufactories such as solar city and detect the connectivity by themselves (utilities).

Mathematically, topology identification can be achieved via voltage estimation [23]. But such an approach will fail in a distribution grid with limited sensor data or relatively frequent topological changes [24]–[27]. There are also methods dedicated for distribution grid based on different assumptions. For example, [28]–[30] assume the availability of all switch locations and search for the right combination. State estimation-based methods [31], [32] and power flow-based methods [33] assume the availability of admittance matrix and infrequent topology change. Unfortunately, these assumptions are improper in newly added or reconfigured distribution network, because neither the knowledge of circuit breakers nor the information of admittance matrix may be available.

Fortunately, smart sensors are continuously being deployed across distribution systems [34], [35], thanks to recent advances in communications, sensing and targeted government investments. Some sensor examples include advanced metering infrastructure (AMI) and load side micro-PMUs (μ-PMUs) [36]. Based on them, statistical methods can be used to correct city network topology [18], [19]. Additionally, private industries are integrating sensing capability into DERs for monitoring purpose, e.g., Solar City's photovoltaic systems [37], commercial and residential charging systems, and in-home appliances such as thermostats. The problem is that such data streams are not integrated or utilized by distribution system operators to improve system performance, e.g., DER topology identification.

In this paper, we propose to utilize such data streams to reconstruct DERs' connectivity for distribution automation [38], [39]. We work on two scenarios. One is the chronic lack of topology state observation. In this case, partial network topology may be unknown or incorrect, so one needs to confirm existing network structure or correct topology errors. The other scenario is to understand the unknown connectivity of newly installed DERs to the existing grid. For example, while the utilities may not own the DER devices, they can try to obtain the sensor data from manufactures such as solar city for connectivity identification.

Specifically, we build a probabilistic graphical model of the power grid to model the dependency between neighbor buses. Subsequently, we formulate the topology identification problem as a probability distance minimization problem via the Kullback-Leibler (KL) divergence metric. We prove that the resulting mutual information-based algorithm can find the optimal topology connection [40] if current injections are assumed to be approximately independent. As such an algorithm relies on sensor data only and computationally inexpensive (no need to consider circuit breaker conditions and system admittance matrix [25]), many plug-and-play devices can be identified correctly. Finally, we generalize the method in the tree network to deal with loops by using conditional mutual information.

The performance of the data-driven method is verified by simulations on the standard IEEE 8- and 123-bus distribution test cases [41]–[43], and Electric Power Research Institute (EPRI) 13, 34, 37, and 2998-bus systems. Two data sets of $123,000$ residential households (PG&E) in the North California and 30 houses in Upper-Austria are used [44]. Simulations are conducted via MATLAB Power System Simulation Package (MATPOWER) [42], [43] and OpenDSS [45]. Simulation results show that, provided with enough historical data, the data-driven topology estimate outperforms the estimates from the traditional approaches.

The rest of the paper is organized as follows: Section II introduces the modeling and the problem of data-driven topology identification. Section III uses a proof to justify the applicability of mutual information-based algorithm for distribution system topology re-configuration. A detailed algorithm is illustrated as well. Section IV evaluates the performance of the new method and Section V concludes the paper.

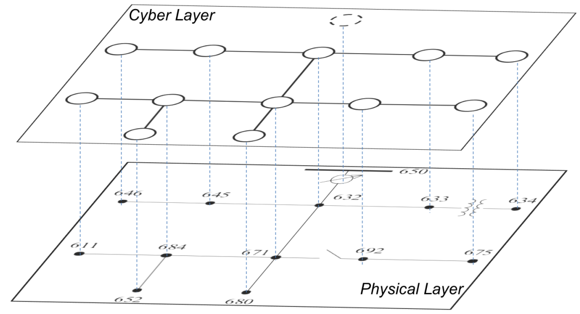

Fig. 1. Cyber-Physical Networks.

II. Probabilistic Modeling of Network Voltages via Graphical Modeling

In this section, we first describe sensor measurements as random variables. With such a definition, we model the distribution grid via a probabilistic graphical model. Based on this modeling, we formally define the problem of data-driven topology identification.

For the modeling, a distribution network is defined as a physical graph $G(\mathcal{V}, \mathcal{E})$ with vertices $\mathcal{V} = \{1, \cdots, n\}$ that represent the buses (PV generator, storage device, and loads), and edges $\mathcal{E}$ that represent their interconnection. It can be visualized as the physical layer in Fig. 1. To utilize the time-series data generated by smart meters, we construct a cyber layer for grid topology reconstruction, where each node is associated with a random variable V_{cyber}. Therefore, a voltage measurement at bus i and time t can be represented as $v_i(t)$.

We use a joint probability distribution to represent the interdependency (edges $\mathcal{E}_{\text{cyber}}$) between different voltage random variables (V_{cyber}).

$$\begin{aligned} p(\mathbf{v}) &= p(v_2, v_3, \dots, v_n) \\ &= p(v_2)p(v_3|v_2)\dots p(v_n|v_2, \dots, v_{n-1}), \end{aligned} \tag{1}$$

where v_i ($i > 1$) represents the voltage measurement at bus i. Bus 1 is a reference bus, so it is fixed as a constant with a unit magnitude and a zero phase angle.

If the quantization of a bus voltage has m levels, one will need to store and manipulate a discrete probability distribution for m^{n-1} values. This is computationally expensive for calculating marginal distribution or maximal joint distribution in large systems. Therefore, we would like to approximate a distribution using a reasonable number of specifying values that capture main dependence [46] (edges $\mathcal{E}$) in the physical layer. For example, we can approximate the true distribution $p(\mathbf{v})$ with another simplified distribution $p_a(\mathbf{v})$. In next section, we will show that such an approximation is exact without approximation error when the current injections are independent.

To approximate a probability distribution $p(\mathbf{v})$ with another probability $p_a(\mathbf{v})$, one can minimize Kullback-Leiber (KL) divergence, which measures the difference between two probability distributions [47].

$$D(p||p_a) = E_{p(\mathbf{v})} \log \frac{p(\mathbf{v})}{p_a(\mathbf{v})}, \tag{2}$$

where p_a is constrained to be with a tree structure or a loopy structure in the work.

Remark We will show later that minimizing the KL divergence between $p(\mathbf{v})$ and $p_a(\mathbf{v})$ is equivalent to maximizing the data likelihood.

A. Problem Definition

The problem of distribution grid topology reconstruction is defined as follows.

- Problem: data-driven topology reconstruction
- Given: a sequence of historical voltage measurements $v_i(t), t = 1, \ldots, T$ and an unknown or partially known grid topology, as shown in Fig. 3
- Find: the local grid topology $\mathcal{E}$ in the dashed box, e.g., Fig. 3, based on $D(p||p_a)$

III. Mutual Information-Based Algorithm for Distribution Grids

While a complete description of a probabilistic graphical model requires a maximum complexity of m^{n-1}, a tree-dependent probabilistic graphical model only requires $m(n-1)$ specifiers, which generates a significant saving due to the Markov property. This is because, under the assumption that any two non-descendant nodes are independent conditioning on their parents, a Bayesian network suffices to determine a probability distribution. Therefore, we can describe the cyber layer relationship as a product of pairwise conditional probability distribution,

$$p_a(\mathbf{v}) = \prod_{i=2}^{n} p(v_i|v_{\text{pa}(i)}), \tag{3}$$

where $v_{\text{pa}(i)}$ is the (random) variable designated as the direct predecessors or parent variable node of v_i in some orientation of the tree. In Fig. 2, we show the mutual information of pairwise current variables. The relatively small mutual information means that one can approximate that the current are independent with some approximation error.

In addition to this intuitional plot, we will show why we can approximate $p(\mathbf{v})$ in (3). Then, we will show an optimal topology identification method based on this assumption. Specifically, we show that if current injections are approximated as independent, we can use historical data to find the optimal approximation distribution $p_a(\mathbf{v})$ of the true distribution $p(\mathbf{v})$. Further, such an approximation leads to a computationally feasible solution for finding the topology in the physical layer.

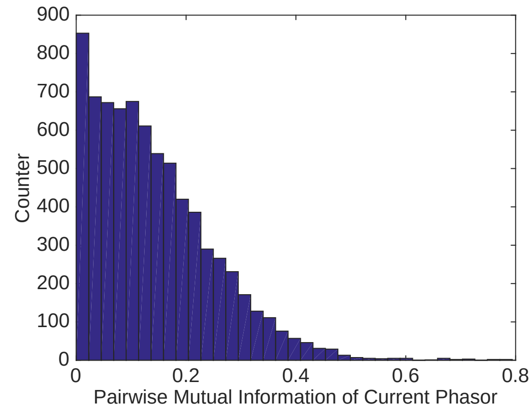

Fig. 2. pairwise mutual information of current phasor.

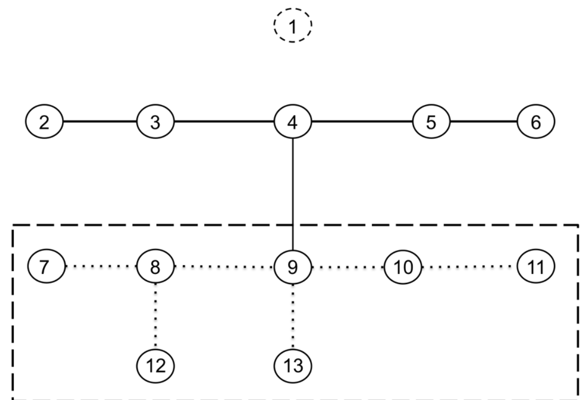

Fig. 3. cyber layer of the 13-bus system.

Lemma 1. *If current injections are approximated as independent, then voltages are conditionally independent in a tree network, given their parent nodes' voltage information.*

Proof: In a distribution system, voltages are usually within the nominal range. Therefore, current injections play the major role in adapting the power injections to balance loads. If users' loads can be approximated as mutually independent, we would like to approximate current injections to be independent for the following derivations.

Let the current injections $I_i \in \mathcal{C}$, $i \in \{1, 2, \cdots, n\}$, be modeled as independent random variables. Let y_{ij} denotes the line admittance between bus i and bus j. Given the reference bus value as $V_1 = v_1$, we can find the relationship between voltages and currents for each bus i in Fig. 4 except node m,

$$V_i = \frac{I_i + v_1 y_{1i}}{y_{ii}}, \quad i = 2, \cdots, n. \tag{4}$$

With the assumption that current injections, e.g., I_i, are independent with others, $V_i|V_1$ and $V_j|V_1$ are independent for $i, j \in 2, \ldots, n$ and $i \neq j$. Now, we analyze a more general tree network with an additional node m connected to bus n, which is shown in dashed line circumference circle of Fig. 4.

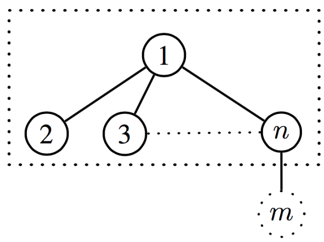

Fig. 4. (n+1)-bus system

The currents and voltages now have the following relationship

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_m \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} & \dots & -y_{1n} & 0 \\ -y_{12} & y_{22} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -y_{1n} & 0 & \dots & y_{nn} & -y_{nm} \\ 0 & \dots & \dots & -y_{nm} & y_{mm} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ V_m \end{bmatrix}.$$

Given $V_1 = v_1$, we have

$$\begin{bmatrix} I_2 + v_1 y_{12} \\ \vdots \\ I_n + v_1 y_{1n} \\ I_m \end{bmatrix} = \begin{bmatrix} y_{22} & 0 & \dots & 0 & 0 \\ 0 & y_{33} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & y_{nn} & -y_{nm} \\ 0 & 0 & \dots & -y_{nm} & y_{mm} \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ \vdots \\ V_n \\ V_m \end{bmatrix}.$$

For bus 2 to bus $n-1$, conditionally independence proof in (4) still holds. But for bus n, it is now connected to bus m. To explore the conditional dependence between bus n and bus $i \in \{2, \cdots, n-1\}$, we use the following derivation.

$$I_n + v_1 y_{1n} = y_{nn} V_n - y_{nm} V_m, \tag{5}$$
$$I_m = -y_{nm} V_n + y_{mm} V_m. \tag{6}$$

Since bus m only connects with bus n, $y_{mm} = y_{nm}$. Further, $y_{nn} = y_{nm} + y_{1n}$ by the definition of admittance matrix. Hence, we can combine (5) and (6).

$$\begin{aligned} I_n + v_1 y_{1n} + I_m &= y_{nn} V_n - y_{nm} V_m - y_{nm} V_n + y_{mm} V_m \\ &= y_{1n} V_n, \\ V_n &= \frac{I_n + I_m + v_1 y_{1n}}{y_{1n}}. \end{aligned} \tag{7}$$

Since the current injections are independent, e.g., $I_i \perp I_j$ for $2 \leq i \leq n-1$ and $j \in n, m$, $I_n + I_m$ and I_i are independent as well. Therefore, $V_n | V_1$ is independent with $V_i | V_1$ due to (4) and (7).

This proof can be extended easily to the case where bus $i \in \{2, \cdots, n-1\}$ is connected to one more bus beside bus 1. In more general cases, where each bus has more than two buses connected, we can aggregate these buses into a single bus and use the proof above to show the conditional independence of voltages. Therefore, we can conclude that $V_i \perp V_j | V_k$, if V_i and V_j share a common parent V_k in the tree network. ■

After showing that $V_i \perp V_j | V_k$, Lemma 2 shows that a mutual information-based maximum weight spanning tree algorithm is able to minimize the approximation error by finding the best-fitted topology in a short time.

Lemma 2. *Recall the KL divergence [47] below*

$$D(p \| p_a) = E_{p(\mathbf{v})} \log \frac{p(\mathbf{v})}{p_a(\mathbf{v})}. \tag{8}$$

If the voltages are conditionally independent, a mutual information-based maximum weight spanning tree algorithm is able to find the best-fitted topology, such that the approximation error (KL divergence) is minimized.

Proof: See Appendix A. ■

With Lemma 1 and Lemma 2, we present our main result in the following.

Theorem 1. *In a radial distribution power grid, mutual information-based maximum spanning tree algorithm finds the optimal approximation of $p(\mathbf{v})$ and its associated topology connection, if current injections are approximated as independent.*

Proof: By using Lemma 1, we know that if the current injections are approximated as independent, then voltages are conditionally independent in a tree network. When the voltages are conditionally independent in a tree structure, by Lemma 2, a mutual information-based maximum weight spanning tree algorithm is able to find the best-fitted topology, such that the approximation error (KL divergence) is minimized. Therefore, we can use such a maximum spanning tree algorithm to find the power grid topology. ■

In Section IV, we will use numerical examples to demonstrate Theorem 1. Specifically, by using the mutual information as the weight, a maximum weight spanning tree finds a highly accurate topology of a distribution grid, making the proposed algorithm suitable for the task of smart grid topology identification with DERs.

A. Why Mutual Information-based Algorithm Works?

One key step to finding the topology is to compare the mutual information. We use the following lemma to illustrate why such a concept is important to find the correct topology.

Lemma 3. *In a distribution network with a tree structure and conditional independence assumption in Theorem 1, $I(V_j, V_i) \geq I(V_j, V_k)$ given $j, k \in pa(i)$, $k \notin r(j)$ and $j \notin pa(k)$.*

Proof: By the definition of mutual information, it has the following chain rule property [47].

$$\begin{aligned} I(V_i, V_j, V_k) &= I(V_i, V_j) - I(V_i, V_j | V_k) \\ &= I(V_j, V_k) - I(V_j, V_k | V_i). \end{aligned}$$

Since $V_j | V_i$ is independent with $V_k | V_i$, the conditional mutual information $I(V_j, V_k | V_i)$ is zero. Then,

$$I(V_i, V_j) = I(V_j, V_k) + I(V_i, V_j | V_k).$$

Due to the fact that mutual information is always non-negative,

$$I(V_j, V_i) \geq I(V_j, V_k).$$

■

This means that a node will always have a larger mutual information than the mutual information between this node and a node further away.

Remark Notice that there is a special case when the tree structure topology is hard to detect by our proposed algorithm. Such a topology is with symmetric topology and loads. For example, let bus 1 has two children: bus 2 and bus 3. If the two branches 1-2 and 1-3 are with the same impedance, and if the loads on bus 2 and bus 3 are the same all the time, then our algorithm will have poor performance. This is because the voltages of bus 2 and bus 3 are the same all the time and this maximizes their mutual information, although they are not connected. However, such an instance requires the same impedance, the same loads, and a symmetric topology. This rarely happens in practice.

In the next, we present the algorithm.

Algorithm 1 Tree Structure Topology Reconstruction

Require: $v_i(t)$ for $i = 2, \dots, n$, $t = 1, \dots, T$

1: **for** $i, j = 2, \dots, n$ **do**
2: Compute mutual information $I(V_i, V_j)$ based on $v_i(t)$.
3: **end for**
4: Sort all possible bus pair (i, j) into nonincreasing order by $I(V_i, V_j)$. Let $\tilde{\mathcal{E}}$ denote the sorted set.
5: Let $\hat{\mathcal{E}}$ be the set of nodal pair comprising the maximum weight spanning tree. Set $\hat{\mathcal{E}} = \emptyset$.
6: **for** $(i, j) \in \tilde{\mathcal{E}}$ **do**
7: **if** cycle is detected in $\hat{\mathcal{E}} \cup (i, j)$ **then**
8: **Continue**
9: **else**
10: $\hat{\mathcal{E}} \leftarrow \hat{\mathcal{E}} \cup (i, j)$
11: **end if**
12: **if** $|\hat{\mathcal{E}}| == n - 2$ **then**
13: **break**
14: **end if**
15: **return** $\hat{\mathcal{E}}$
16: **end for**

Step 6-16 build a maximum spanning tree using pairwise mutual information as the weight. This algorithm is modified from the well-known Kruskal's minimum weight spanning tree algorithm [48], [49], which has a running time of $O((n-2)\log(n-1))$ for a radial distribution network with n buses. Therefore, the proposed algorithm can efficiently reconstruct the topology with low computational complexity.

B. Adaptation for Distribution Grid with a Loop

In the previous subsection, we show a mutual information-based algorithm able to identify tree-structure topology. In practice, a ring structure exists for robustness, e.g., Fig. 5 [18], [19]. In a ring structure, there must be a node with two neighbors. In a graphical model, this means that there is one child with two parents. Similar to the previous section, probability distribution p_a can be written as

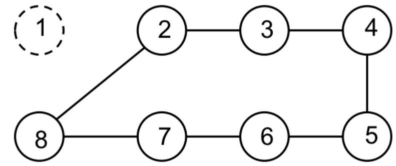

Fig. 5. loop network of 8 buses.

$$p_a(\mathbf{v}) = p(v_n | v_{\text{pa}(n),1}, v_{\text{pa}(n),2}) \prod_{i=2}^{n-1} p(v_i | v_{\text{pa}(i)}),$$

where $\{\text{pa}(n), 1\}$ and $\{\text{pa}(n), 2\}$ represent the two parent nodes of V_n. Then, we have

$$\begin{aligned}
D(p\|p_a) &= \sum p(\mathbf{v}) \log \frac{p(\mathbf{v})}{p_a(\mathbf{v})} \\
&= \sum p(\mathbf{v}) \log p(\mathbf{v}) - \sum p(\mathbf{v}) \sum_{i=2}^{n-1} \log \frac{p(v_i, v_{\text{pa}(i)})}{p(v_i)p(v_{\text{pa}(i)})} \\
&\quad - \sum p(\mathbf{v}) \log \frac{p(v_n, v_{\text{pa}(n),1}, v_{\text{pa}(n),2})}{p(v_n)p(v_{\text{pa}(n),1}, v_{\text{pa}(n),2})} \\
&\quad - \sum p(v_i) \sum_{i=2}^{n} p(v_i).
\end{aligned}$$

Therefore,

$$\begin{aligned}
D(p\|p_a) = &-\sum_{i=2}^{n-1} I(V_i; V_{\text{pa}(i)}) - I(V_n; V_{\text{pa}(n),1}, V_{\text{pa}(n),2}) \\
&+ \sum_{i=2}^{n} H(V_i) - H(V_2, \cdots, V_n).
\end{aligned} \tag{9}$$

Thus, for each extended mutual information $I(V_n; V_{\text{pa}(n),1}, V_{\text{pa}(n),2})$, we search for maximum weighted spanning tree and compute the total mutual information $\sum_{i=2}^{n-1} I(V_i; V_{\text{pa}(i)}) + I(V_n; V_{\text{pa}(n),1}, V_{\text{pa}(n),2})$. Then, one compares the total mutual information and chooses the largest one. Finally, one can further generalize the proof to include more than one loop.

To summarize our algorithm for the simulation, we use the flow chart in Fig. 7 and the algorithm description in **Algorithm 2**.

Algorithm 2 Topology Reconstruction with a Loop

Require: $v_i(t)$ for $i = 2, \ldots, n$, $t = 1, \ldots, T$
if there is a loop in the network **then**
  Compute $I(V_n; V_{\text{pa}(n),1}, V_{\text{pa}(n),2})$ with all possible values of n.
  Remove the branch with the maximum value so that the remaining network forms a tree.
end if
for $i, j = 2, \ldots, n-1$ **do**
  Compute the mutual information $I(V_i, V_j)$ based on $v_i(t)$.
end for
Sort all possible bus pair (i, j) into non-increasing order by $I(V_i, V_j)$. Let $\tilde{\mathcal{E}}$ denote the sorted set.
Let $\hat{\mathcal{E}}$ be the set of nodal pair comprising the maximum weight spanning tree. Set $\hat{\mathcal{E}} = \emptyset$.
for $(i, j) \in \tilde{\mathcal{E}}$ **do**
  if cycle is detected in $\hat{\mathcal{E}} \cup (i, j)$ **then**
    Continue
  else
    $\hat{\mathcal{E}} \leftarrow \hat{\mathcal{E}} \cup (i, j)$
  end if
  if $|\hat{\mathcal{E}}| == N - 3$ **then**
    break
  end if
  return $\hat{\mathcal{E}}$ with the detected loopy branch if there is any
end for

Due to the loop structure, the computational cost involved in (9) is $O(n(n-1)log(n-1))$.

C. Adaptation for Smart Meter with Voltage Magnitude Data

As smart meters are only loosely synchronized, accurate phase angle information is hard to obtain for mutual information computation. Fortunately, the angle variance is small in a distribution grid. Therefore, we can use the chain rule of mutual information below for an approximation of $I(V_i; V_j)$ when only the voltage magnitude measurements are available. In order to analyze such an approximation, we decompose the mutual information into four terms and compare their contributions to $I(V_i; V_j)$.

$$\begin{aligned} I(V_i; V_j) &= I\big(|V_i|, \angle V_i; |V_j|, \angle V_j\big) \qquad (10) \\ &= I\big(|V_i|; |V_j|, \angle V_j\big) + I\big(\angle V_i; |V_j|, \angle V_j| \mid |V_j|\big) \\ &= I\big(|V_i|; |V_j|\big) + \underbrace{I\big(|V_i|; \angle V_j \mid |V_i|\big)}_{A} \\ &+ \underbrace{I\big(\angle V_i; |V_j| \mid |V_i|\big)}_{B} + \underbrace{I\big(\angle V_i; \angle V_j \mid |V_i|, |V_j|\big)}_{C} \end{aligned}$$

Fig. 6 shows a numerical comparison of all possible pairwise mutual information and its sub-components in an IEEE 123-bus simulation. In Fig. 6, terms A, B, and C are relatively small when comparing to $I\big(|V_i|; |V_j|\big)$. This is because voltage phase angles are with very small changes in the distribution grid, thus has less information than the voltage magnitudes. Subsequently, one can use $I\big(|V_i|; |V_j|\big)$ for comparison in the topology identification process instead of using $I(V_i; V_j)$. In next section, we will verify this idea, where we test (a) when the phase angle is available and (b) when such information is unavailable.

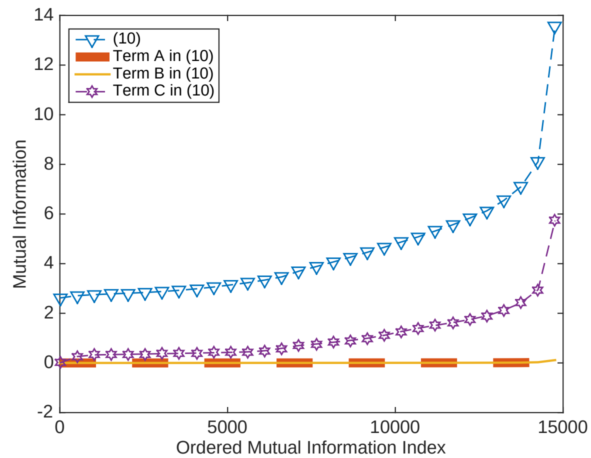

Fig. 6. pairwise mutual information.

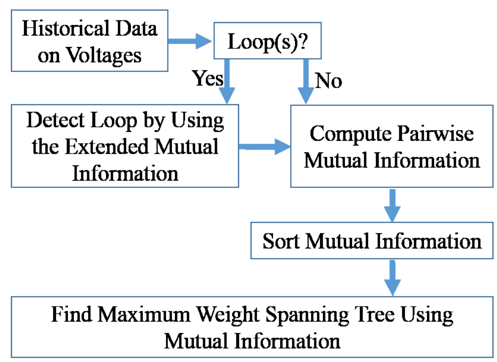

Fig. 7. flow chart of the proposed algorithm.

In Fig. 7, we present the flow chart for subsequent simulations.

D. Limitations of the Method

There are inherent limitations of a data-driven approach. For example, the proposed method may not work properly when (1) there is no historical data, (2) the power network is highly meshed with many loops (computationally expensive), (3) the recorded data is with low quality (e.g., quantized data for billing purpose), and (4) the loads are identical at all nodes. While (4) is rare, it happens when the nodes in a network are all with the same solar panels, but without power consumptions. This causes the currents to be highly similar to each other, violating our assumption in Lemma 1.

IV. Simulations

The simulations are implemented on the IEEE PES distribution networks for IEEE 8-bus and 123-bus systems [41], [50], Electric Power Research Institute (EPRI) 13, 34, 37, and 2998-bus systems. In each network, feeder bus is selected as the slack bus. The historical data have been preprocessed by the MATLAB Power System Simulation Package (MATPOWER) [42], [43] and OpenDSS [45]. To simulate the power system behavior in a more realistic pattern, the load profiles from Pacific Gas and Electric Company (PG&E) and "ADRES-Concept" Project of Vienna University of Technology [44] are adopted as the real power profile in the subsequent simulation. PG&E load profile contains hourly real power consumption of $123,000$ residential loads in the North California, USA. 'ADRES-Concept" Project load profile contains real and reactive powers profile of 30 houses in Upper-Austria. The data were sampled every second over 14 days [44]. For example, Fig. 8 shows the data set from "ADRES-Concept" Project.

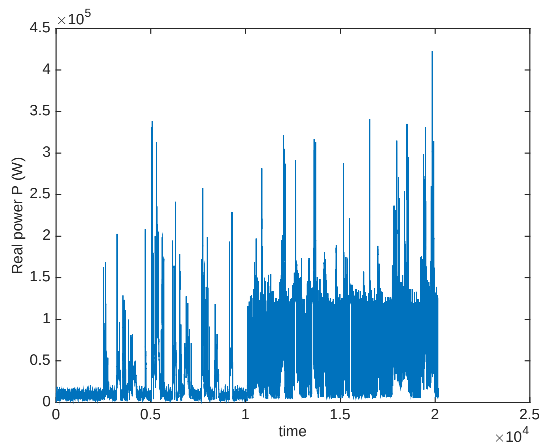

Fig. 8. 1-minute power profile.

For PG&E data set, reactive powers are not available. For its reactive power q_i at bus i, we simulate it with three different scenarios:

- *Random Reactive Power:* $q_i(t) \sim \text{Unif}(0.5\mu_i^q, 1.5\mu_i^q)$, $t = 1, \cdots, T$, *where the mean* $\mu_i^q \in (60W, 180W)$ *is given in the IEEE PES distribution network;*
- *Random Power Factor:* $q_i(t) = p_i(t)\sqrt{1 - pf_i(t)^2} / pf_i(t)$, *where* $pf_i(t) \sim \text{Unif}(0.85, 0.95)$*;*
- *Fixed Power Factor:* $q_i(t) = p_i(t)\sqrt{1 - pf_i^2}/pf_i$, *where the fixed power factor* $pf_i \sim \text{Unif}(0.85, 0.95), \forall i \in \mathcal{V}$.

A. Tree Networks without DERs

1) PG&E Systems: To obtain voltage measurements at time t, i.e., $|v_i(t)|$ and $\theta_i(t)$ for tree networks without DERs, we run a power flow based on the power profile above. Therefore, time-series data are obtained by repeatedly running the power flow to generate hourly data over a year. Totally, $T = 8760$ measurements are obtained at each bus for PG&E data set. Finally, these voltage measurements are used for the mutual information-based algorithm.

To simplify the analysis, we model V_i at bus i as a two-dimensional real Gaussian random vector, instead of a complex random variable. For simulations over tree networks (IEEE 123-bus in Fig. 9) without DERs, the first step of our proposed algorithm is to compare mutual information between different pairs. Fig. 10 displays pairwise mutual information of bus 26 and bus 109. We can see that the mutual information of a connected branch is much larger when compared with other pairs. This confirms Lemma 3.

To demonstrate the detection result, a heat map of the mutual information matrix is shown in Fig. 11, where bus $90, \cdots, 115$ are included. In this figure, a circle represents the true connection while a crossing indicates a detected connection between the row and column indexes. Therefore, if a circle is superposed by a crossing, a correct topology identification is claimed. We observe that

1) the diagonal element has the largest mutual information in each row because it is the self-information [47].
2) the coordinate associated with the true branch has the largest mutual information in each row (excluding the diagonal element). This fact illustrates that using the pairwise mutual information as the weight for topology detection is consistent with the physical connectivity.

We repeat the simulation above in different setups and summarize the averaged performances in Table I, where detection error rate is defined as $(1 - \frac{\sum_{i,j\in\mathcal{E}} \mathbb{I}\left((i,j)\in\hat{\mathcal{E}}\right)}{|\mathcal{E}|}) \times 100\%$. Here, $\hat{\mathcal{E}}$ denotes the estimated set of branches and $|\mathcal{E}|$ denotes the size of the set $\mathcal{E}$. Table I summarizes detection error rate on IEEE 8-bus and 123-bus systems. It shows that the proposed algorithm can recover both systems without error with repeated simulations under three setups.

TABLE I. Detection Error Rate

	8-Bus Network	123-Bus Network
Random Reactive Power	0%	0%
Random Power Factor	0%	0%
Fixed Power Factor	0%	0%

In addition to detection error rate for a fixed number of unknown topology, we also simulate cases when the number of unknown topology is changing, e.g., Fig. 12. We compare the proposed algorithm with a traditional algorithm (requiring admittance matrix) and the algorithm in [25], referred as "Bolognani-Schenato Algorithm". The x-coordinate represents the number of edges needed to be identified. The y-coordinate represents the detection error rate. As the number of unknown edges increases, our approach consistently has a zero error rate, while the other methods' detection ability decreases. The failure of both methods is likely due to their assumptions, such as known admittance matrix, a fixed inductance/resistance ratio, or sufficiently large nominal voltage.

2) EPRI Systems: To explore the performance of our algorithm on a large-scale network, we use a radial distribution grid provided by Electric Power Research Institute (EPRI). This grid contains one feeder with 2998 buses. The detected topology is shown in Fig. 13. The detection error is shown in Table II.

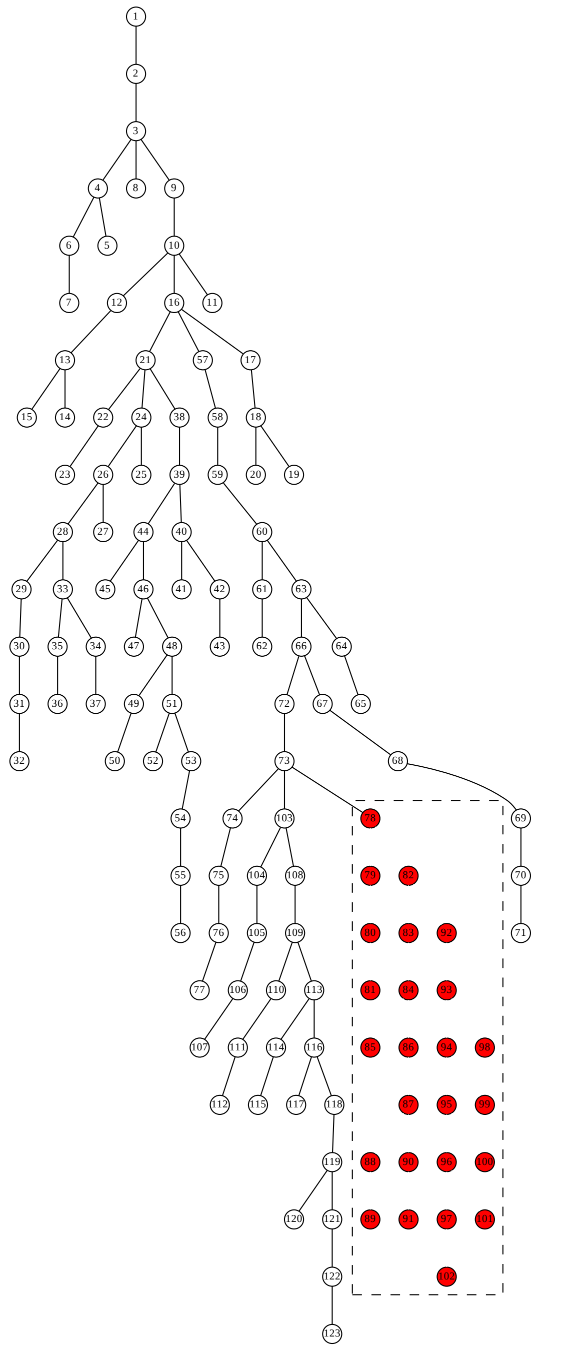

Fig. 9. IEEE 123-bus system: In the dashed box, the topology is unknown.

TABLE II. EPRI SYSTEM DETECTION ERROR RATE

Bus Number	Error Rate
13	0%
34	0%
37	0%
2998	0%

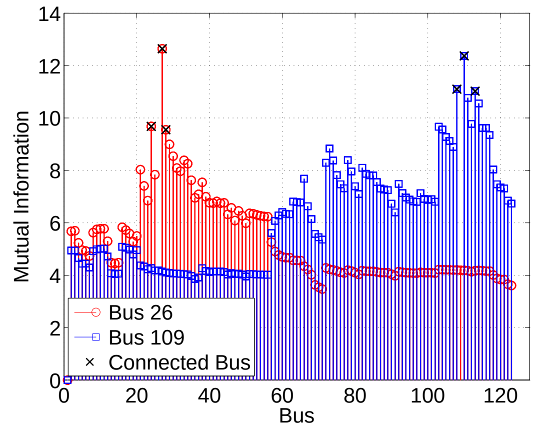

Fig. 10. pairwise mutual information

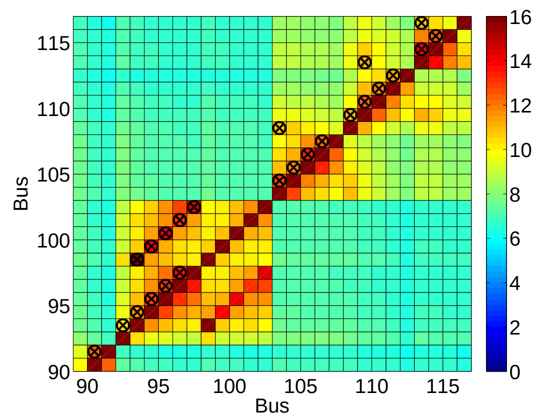

Fig. 11. heat map of the mutual information matrix. Black circle: true branches in the 123-bus network. Black crossing: detected branches.

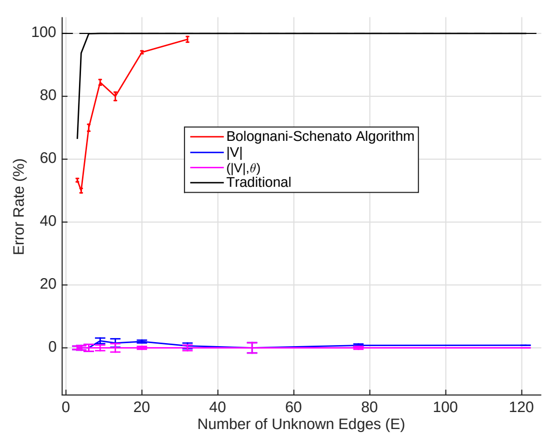

Fig. 12. error comparison.

B. *Tree Networks with DERs*

In the simulation of tree networks with DERs, solar panels are selected as the source of renewable energy. The hourly power generation profile is computed by using PVWatts Calculator, an online application developed by the National Re-

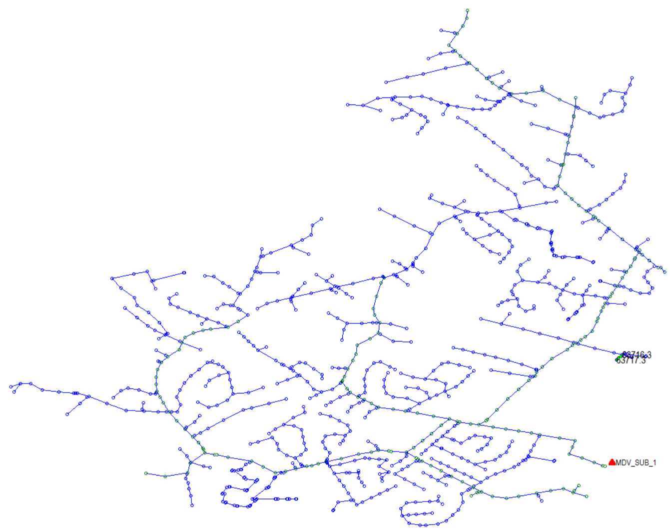

Fig. 13. detected EPRI 2998-bus topology.

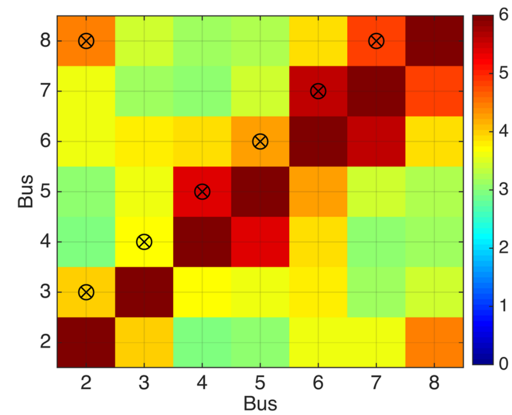

Fig. 14. heat map of the mutual information matrix. Black circle: True branches in the 8-bus network. Black crossing: The detected branches.

newable Energy Laboratory (NREL) that estimates the power generation of a photovoltaic system based on weather and physical parameters [51]. The hourly data are computed based on the weather history of Mountain View, CA, USA and the physical parameters of a solar panel with a capacity of 5 kW.

We randomly choose 12 load buses in the 123-bus system to have solar panels. The hourly generation profile at bus i is modeled as a Gaussian distribution, $p_i^r(t) \sim \mathcal{N}(P^r(t), 0.05)$, where $t = 1, \cdots, T$ and $P^r(t)$ denotes the power generation computed by PVWatts with a unit of kW. Finally, the renewable power generator is modeled as a negative load. After simulations, we obtain similar mutual information plots like Fig. 10 and Fig. 11. Finally, Table III summarizes the detection error rate on 123-bus system demonstrating the robustness of our algorithm to renewables.

TABLE III. DETECTION ERROR RATE WITH RENEWABLE ENERGY GENERATORS ON THE 123-BUS SYSTEM

	V	$\|V\|$
Random Reactive Power	0%	2.44%
Random Power Factor	0%	0.81%
Fixed Power Factor	0.81%	0.81%

C. Networks with a Loop

When adding a loop to the network, we apply adapted mutual information-based algorithm in **Algorithm 2** and achieves 100% accuracy in the network without renewable and near 100% accuracy in the network with renewable. For example, Fig. 14 shows the heat map of the mutual information matrix for a loopy system with 8 buses shown in Fig. 5. All of the topology connections were detected correctly.

D. Algorithm Sensitivities

1) Sensitivity to historical data length: To explore the sensitivity of the proposed algorithm to the sample number, we run Monte Carlo simulations by using data from 10 days to 300 days. The results are summarized in Fig. 15. We observe that when more than 30 days' observations are available, our algorithm can stably reconstruct the topology with historical data. This result reflects that our algorithm can provide robust reconstruction with a short period of historical data.

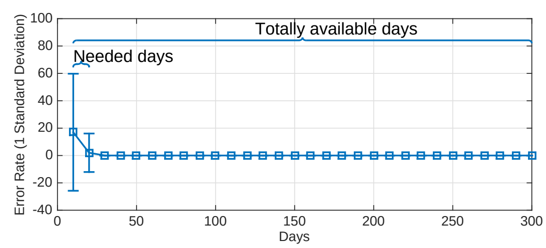

Fig. 15. error and its confidence interval versus the number of observations.

2) Sensitivity to the data resolution: In this simulation, we aggregate data points from [44] every 1 minute, 30 minutes, and 60 minutes. We generate the voltage profiles using IEEE 123-bus test case and assume the topology between bus 78 and bus 102 is unknown. Fig. 16 shows the simulation results. When data are sampled at 1-minute interval, we need about 3 hours voltage profile to reconstruct the topology perfectly. As discussed in [24], the distribution grid usually reconfigures every 3 hours. Therefore, the proposed algorithm can reconstruct the topology in real-time. If the sampling period is 30 minutes or 60 minutes, about 30 hours' data are required to reconstruct the topology [52]. Therefore, the minimum computational time can be shorter when data with higher sampling rate is available.

Remark In addition to data resolution, one also needs to consider the case when the topology changes while the historical data are acquired. If the acquired historical data include data from two different topologies, our proposed algorithm will have poor performance. However, our algorithm can be executed repeatedly with low cost. Thus, we suggest using our

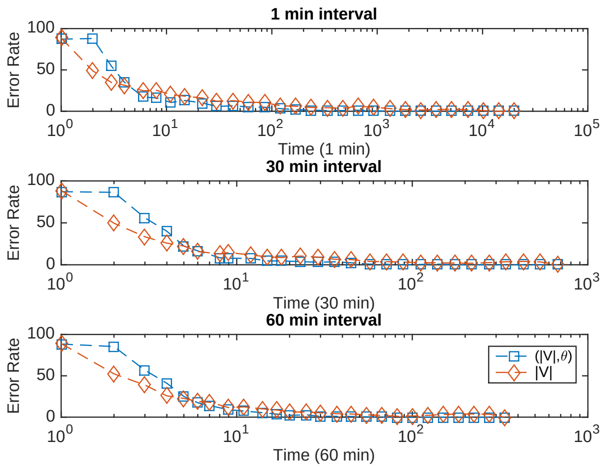

Fig. 16. reconstruction error rates with different data resolutions.

result when several detection results at consecutive time slots return the same topology.

3) Sensitivity to the Data Coming from Different Seasons: To understand our algorithm's sensitivity to the different days, we compare data streams taken in two different seasons of the year. This simulation result is shown in Fig. 17. As one can see, data coming from different times of the year do not affect the algorithm.

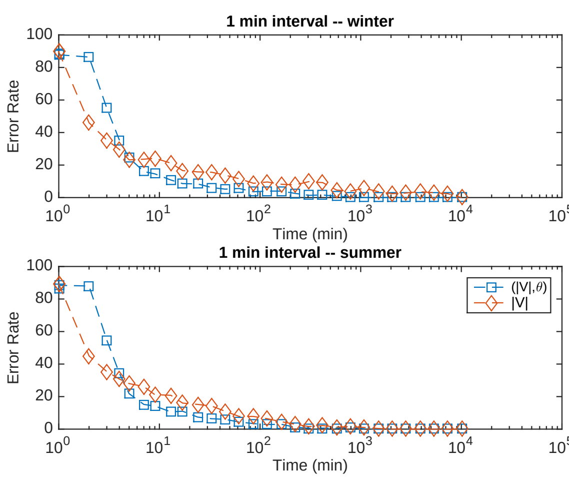

Fig. 17. reconstruction error rates with data coming from different seasons.

4) Sensitivity to the Smart Meter Accuracy: The proposed method depends on the sensor measurements. So, it is important to know the existing meters' accuracy and whether such an accuracy is good enough for the proposed method. Therefore, we simulate the sensor error rate of 0.01% in [53]. For comparison purpose, we also simulate the error of 0.05%. Fig. 18 shows the reconstruction error rates with different noise levels. Therefore, the proposed algorithm can provide good topology identification rate with the current sensor noise level.

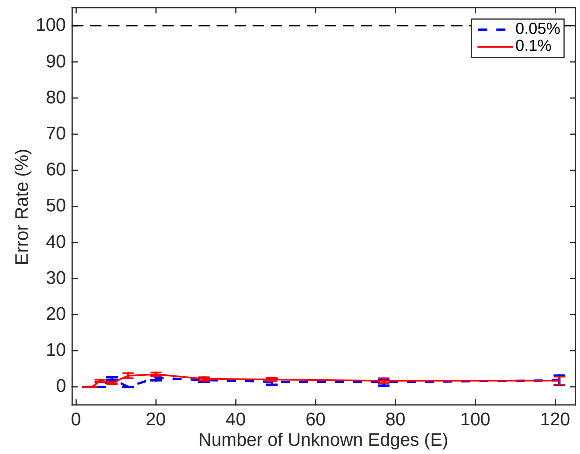

Fig. 18. reconstruction error rates with different noise levels.

Notice that if the smart meters data's accuracy is reduced, e.g., rounded up, the statistical relationship may be lost. And our algorithm may have poor performance.

5) Sensitivity to Flat Loads: One may expect that a diversity in the historical data to be important. Therefore, if the load profiles are flat, the proposed approach can fail. To check for it, we simulate the flat load scenario, where we use a fixed load plus a Gaussian noise with a variance of 1% times the load value. This leads to Fig. 19, which represents the simulated loads for bus 3 in the 123-bus.

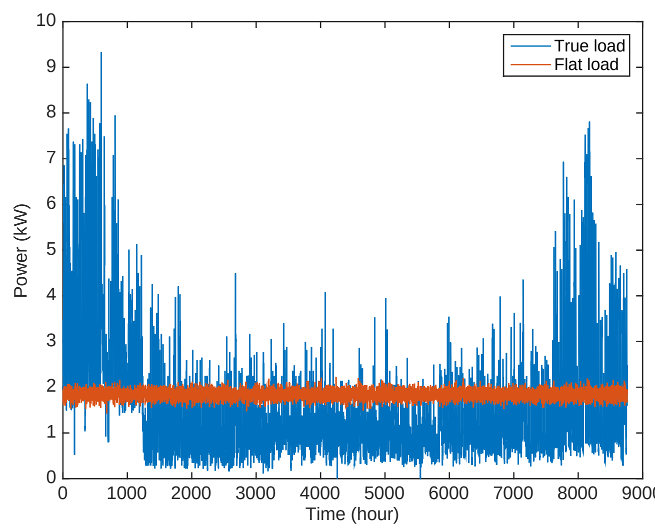

Fig. 19. flat load

Table IV shows the simulation result. We observe that if the load profiles are flat, the proposed approach still works. This is

TABLE IV. DETECTION ERROR RATE WITH RENEWABLE ENERGY GENERATORS ON THE 123-BUS SYSTEM WITH FLAT LOADS

	V	$\|V\|$
Random Reactive Power	0%	0%
Random Power Factor	0%	0%
Fixed Power Factor	0%	0%

because the difference between different loads comes from the "noise". And these noisy are independent of each other, making the current injections independent as well. When the currents injections are independent, the assumption of our theorem is completely satisfied. This makes our algorithm achieving even better simulation result without error.

6) Sensitivity to the Redundancy of Available Information: Nowadays, smart meters collect real-time measurements of voltage magnitude. Without μ-PMU being widely adopted, we want to explore how the proposed algorithm performs on the voltage magnitude measurements only. Therefore, we model the voltage magnitude $|V_i|$ at bus i as a real Gaussian random variable, $|V_i| \sim \mathcal{N}(\mu_{|V_i|}, \Sigma_{|V_i|})$, where $\mu_{|V_i|}$ denotes the mean and $\Sigma_{|V_i|}$ denotes the variance. Both parameters are learned from historical observations.

TABLE V. DETECTION ERROR RATE WITH ONLY VOLTAGE MAGNITUDE

	8-Bus Network	123-Bus Network
Random Reactive Power	0%	0.81%
Random Power Factor	0%	1.63%
Fixed Power Factor	0%	2.44%

Table V show the SDR on 8-bus and 123-bus systems by utilizing voltage magnitude only. For the 8-bus system, the proposed algorithm can still achieve zero error. For the 123-bus system, detection error rate is still low, which is about 2.44%. Notice that our setup is in the extreme case where no branch connectivity is known at all. In practice, at least, a partial network structure is known. For example, if 57 buses (selected randomly) out of 123 buses are unknown, our simulation result shows that the proposed algorithm can always reconstruct the topology correctly by only using voltage magnitudes.

V. CONCLUSION

In order to identify distributed energy resources' connectivity, we propose a data-driven approach based on sensor measurements. Unlike existing approaches, which require the knowledge about circuit breakers or the admittance matrix, our proposed algorithm relies on smart metering data only. We formulate the topology reconstruction problem as a joint distribution (voltage phasors) approximation problem under the graphical model framework. A mutual information-based maximum weight spanning tree algorithm is proposed based on our proof of how to minimizing Kullback-Leibler divergence in a distribution system. Moreover, we extend our algorithm from tree structures to loopy structures to generalize the usage. Finally, we simulate the proposed algorithm on IEEE 8- and 123-bus systems and EPRI 13-, 34-, 37-, and 2998-bus systems, compare with past methods, and observe highly accurate detection results in cases with and without distributed energy resources.

ACKNOWLEDGMENT

We would like to thank Roger Dugan (EPRI), the author for OpenDSS, and Robert Entriken (EPRI) for simulations over relatively large EPRI distribution systems. We would also like to thank Vienna University of Technology - Institute of Energy Systems and Electrical Drives for providing the ADRES-Concept data set.

APPENDIX

Proof: To minimize the distance between $p_a(\mathbf{v})$ and $p(\mathbf{v})$, we recall the definition of KL divergence [47].

$$\begin{aligned} D(p\|p_a) &= E_{p(\mathbf{v})} \log \frac{p(\mathbf{v})}{p_a(\mathbf{v})} \\ &= \sum p(\mathbf{v}) \log \frac{p(\mathbf{v})}{p_a(\mathbf{v})} \\ &= \sum p(\mathbf{v}) \big(\log p(\mathbf{v}) - \log p_a(\mathbf{v}) \big) \\ &= \sum p(\mathbf{v}) \log p(\mathbf{v}) - \sum p(\mathbf{v}) \log p_a(\mathbf{v}). \end{aligned}$$

As $V_i \perp V_j | V_k$, we have $p_a(\mathbf{v}) = \prod_{i=2}^{n} p(v_i | v_{\text{pa}(i)})$. Then,

$$\begin{aligned} D(p\|p_a) &= \sum p(\mathbf{v}) \log p(\mathbf{v}) - \sum p(\mathbf{v}) \log \prod_{i=2}^{n} p(v_i | v_{\text{pa}(i)}) \\ &= \sum p(\mathbf{v}) \log p(\mathbf{v}) - \sum p(\mathbf{v}) \sum_{i=2}^{n} \log p(v_i | v_{\text{pa}(i)}), \end{aligned}$$

where $p(v_2|v_1) = p(v_2)$ due to the fact that v_1 is a constant. By following the definition of conditional probability and by adding $p(v_i)$ into the denominator, we have the following equality

$$\begin{aligned} D(p\|p_a) &= \sum p(\mathbf{v}) \log p(\mathbf{v}) - \sum p(\mathbf{v}) \Bigg[\sum_{i=2}^{n} \log \frac{p(v_i, v_{\text{pa}(i)})}{p(v_i) p(v_{\text{pa}(i)})} \\ &\quad + \sum_{i=2}^{n} \log p(v_i) \Bigg] \\ &= \sum p(\mathbf{v}) \log p(\mathbf{v}) - \sum p(\mathbf{v}) \sum_{i=2}^{n} \log \frac{p(v_i, v_{\text{pa}(i)})}{p(v_i) p(v_{\text{pa}(i)})} \\ &\quad - \sum p(v_i) \sum_{i=2}^{n} \log p(v_i), \end{aligned}$$

where the last equality holds because

$$\begin{aligned} \sum p(\mathbf{v}) \sum_{i=2}^{n} \log p(v_i) &= \sum p(v_i) \sum_{i=2}^{n} \log p(v_i) \\ &= \sum_{\mathbf{v} \backslash v_i} p(\mathbf{v}|v_i) \sum p(v_i) \sum_{i=2}^{n} \log p(v_i) \\ &= \sum p(v_i) \sum_{i=2}^{n} \log p(v_i), \end{aligned}$$

where the notation $\mathbf{v} \backslash v_i$ means all variables in $\mathbf{v}$ except v_i.

Because of the same reason, we have

$$D(p\|p_a) = \sum p(\mathbf{v}) \log p(\mathbf{v}) - \sum p(v_i, v_{\text{pa}(i)}) \cdot \sum_{i=2}^{n} \log \frac{p(v_i, v_{\text{pa}(i)})}{p(v_i)p(v_{\text{pa}(i)})} - \sum p(v_i) \sum_{i=2}^{n} \log p(v_i).$$

Therefore, we observe the following two terms in the KL-divergence expression:

$$H(V_i) = -\sum p(v_i) \sum_{i=2}^{n} p(v_i),$$
$$I(V_i; V_{\text{pa}(i)}) = \sum p(v_i, v_{\text{pa}(i)}) \sum_{i=2}^{n} \log \frac{p(v_i, v_{\text{pa}(i)})}{p(v_i)p(v_{\text{pa}(i)})}. \quad (11)$$

Subsequently,

$$D(p\|p_a) = -\sum_{i=2}^{n} I(V_i; V_{\text{pa}(i)}) + \sum_{i=2}^{n} H(V_i) - H(V_1, \cdots, V_n).$$

Thus, to minimize the KL-divergence between $p_a(\mathbf{v})$ and $p(\mathbf{v})$, we can choose the $n-2$ acyclic edges that maximize the $\sum_{i=2}^{n} I(V_i; V_{\text{pa}(i)})$. This is because $\sum_{i=2}^{n} H(V_i) - H(V_2, \cdots, V_n)$ is irrelevant with the topology structure of distribution grid. Such an algorithm is also known as Chow-Liu algorithm, which has been shown as an optimal algorithm that finds the best product approximation of $p(\mathbf{v})$ [40]. It uses the mutual information of all possible bus pairs within the network and finds maximum weight spanning tree that maximizes the overall mutual information. It is equivalent to minimum KL-divergence in our case. ■

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