

Urban Distribution Grid Line Outage Identification

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Abstract—The growing integration of distributed energy resources (DERs) in urban distribution grids raises various reliability issues due to complex uncertainties. With the large-scale penetration of DERs, traditional outage detection methods, which rely on customers making phone calls and smart meters’ “last gasp” signals, will have limited performance because 1) the renewable generators can supply powers after line outages, and 2) many urban grids are mesh and line outages do not affect power supply. To address these drawbacks, we propose a new data-driven outage monitoring approach based on the stochastic time series analysis with the newly available smart meter data utilized. Specifically, based on the power flow analysis, we prove that the statistical dependency of time-series voltage measurements has significant changes after line outages. Hence, we use the optimal change point detection theory to detect and localize line outages. As the existing change point detection methods require the post-outage voltage distribution, which is unknown in power systems, we propose a maximum likelihood method to learn the distribution parameters from the historical data. The proposed outage detection using estimated parameters also achieves the optimal performance. Simulation results show highly accurate outage identification in IEEE standard distribution test systems with and without DERs using real smart meter data.

I. INTRODUCTION

One of the key goals in building a Smart City is to be able to deliver reliable, economic and sustainable energy to its end users. To achieve this goal, photovoltaic (PV) power devices (renewable generation), energy storage devices, and electric vehicles have been integrated into the distribution grid [1]. Such large-scale integration and deployment of these distributed energy resources (DERs) not only provide more sustainable energy sources but also reduce the electricity cost and transmission loss.

While DERs in principle offer substantial benefits in transition to a sustainable electric grid, successful and reliable integration of these energy resources poses fundamental challenges in system operations. On the generation side, the reverse power flow can render the existing protective systems inadequate. Without appropriate monitoring and control, even a small-scale DER integration could destabilize the local grid and cause reliability issues for customers [2]. On the demand side, the frequent plug-and-charge electric vehicles will impact the distribution grid power quality such as voltage unbalance and transformer overload [3]. These uncertainties can lead to power outages or blackouts in distribution grids, which may cause a loss of thousands to millions of dollars within one-hour [4]. Therefore, a highly active and accurate fault diagnosis process is necessary for distribution grid operation.

In transmission grids, the power outage detection has

received a surge of interest in the past decade. Most of these approaches use the DC power flow approximation and the recently available phasor measurement units (PMU) measurements. For example, changes in phase angles across all buses are compared with potential fault events in [5]. In [6], a transmission grid is formulated as a graphical model and phase angles are used to track the grid topology change. A regularized linear regression is employed to detect power outages in [7]. The approach in [8] compares the branch admittance before and after outages. These methods, however, cannot be directly implemented for the distribution grid because of the following reasons. Firstly, the DC approximation has poor performance in distribution grids as many networks have non-negligible line loss. Secondly, the phase angles are difficult to obtain since micro-PMUs have not been widely deployed in distribution grids. Thirdly, the topology information is unavailable or inaccurate in distribution grids, because many DERs do not belong to the utilities and their connectivities are unknown to the system operators [9].

In distribution grids, the traditional approach to power outage assessment relies on customers making phone calls to report an incident to the Customer Information System, from which the Outage Management System obtains the information and dispatches crews to the field to identify outage areas. They often receive delayed and imprecise outage information, making outage detection and power restoration slow and inefficient. Recent deployment of advanced metering infrastructure (AMI) enables smart meters to send a “last gasp” message when there is a loss of power [10]. In addition, the fault location, isolation, and service restoration (FLISR) technologies and systems have been adopted by the utilities in the U.S., which can further reduce the impact and duration of power interruptions [11].

With the integration of DERs, the approaches above will have limited performance. For example, if rooftop solar panels are installed at the customers premises, the customer can still receive power from renewable generators when there is no power flow in the distribution circuit connecting to the premises. So the (AMI) smart meter at the customer premises cannot detect a power outage. Also, in metropolitan areas, such as New York City, Chicago, and San Francisco, the secondary distribution grids are mesh networks [12]. Hence, a branch outage will not necessarily cause a power outage. However, the distribution grid system operators still need to detect, localize, and identify the out-of-service branches.

Fortunately, the smart meters installed at households enable a new opportunity to utilize the time-series data, which were previously unavailable in the electric power industry [13], to tackle the distribution grid outage detection challenge. One

way to describe the historical data is using random variables. If we model the measurement at each bus as a random variable, a distribution grid can be represented as a multivariate probability distribution. As will be clear later in this paper, a line outage will lead to a change of the statistical dependence between buses, and consequently, a change of the joint distribution. Hence, the outage can be discovered by detecting the change of the multivariate probability distribution. A well-known method to sequentially detect the probability distribution change is change point detection method [14]. In change point detection, the detector observes a sequence of data and reports an outage when it detects a change of data probability distribution due to some events at an unknown time. The objective is to identify an outage as quickly as possible subject to a fixed probability of false alarm [15]. Notably, change point detection method has been applied to a variety of fields, such as computer networks [16], manufacturing quality control [17], structural health monitoring [18], [19], and authorship analysis [20]. Also, change point detection method has been adopted to detect the line outage in transmission grids [8], [21].

The change point detection methods above require the post-outage probability distribution. However, in practice, this probability distribution is hard to obtain in distribution grids because the number of possible post-outage distributions increases exponentially with the growth of the grid size. To overcome this drawback, we propose a maximum likelihood method to learn the unknown post-outage probability distribution parameters from the data. However, change point detection method discussed above can only indicate the time of outages. In the system operation, it is also interesting to localize the line outage. For this localization problem, we firstly prove that the voltages of two disconnected buses are conditionally independent, which is subsequently used to find the line outage without knowing the post-outage probability distribution.

In this paper, we propose a data-driven and optimal algorithm to detect and identify line outages in the distribution grid with DERs. We model a distribution grid as a joint probability distribution. By exploiting the statistical properties of voltage measurements obtained from smart meters, we propose a real-time power outage detection algorithm based on the optimal change point detection method. Unlike existing approaches, this algorithm does not require the knowledge of the post-outage probability distribution. After an outage is detected, we utilize the conditional independence between the nodal voltage measurements to localize the line outage. This outage localization method is scalable for a large distribution network. The performance of our data-driven outage detection and localization algorithm is verified by simulations on the standard IEEE 8- and 123-bus distribution test cases [22] with DERs. Real smart meter data collected from 110,000 Pacific Gas and Electric Company (PG&E) customers are utilized for simulation with different outage scenarios.

The rest of the paper is organized as follows: Section II introduces the modeling and the problem of the data-driven power outage detection and localization. Section III uses a proof to justify that the outage can be detected by change point detection method. Section IV presents the outage localization method. A detailed algorithm for outage detection and localization is illustrated as well. Section V evaluates the performance of the new method and Section VI concludes the paper.

II. SYSTEM MODEL

In order to formulate the power outage detection problem, we need to describe the distribution grid and its data. A distribution grid is defined as a physical network with buses and branches that connect buses. For a distribution grid with M buses, we use $\mathcal{S} = \{1, 2, \dots, M\}$ to represent the set of all bus indices. To utilize the time series data provided by smart meters, the voltage measurement at bus i is modeled as a random variable V_i . We use $\mathbf{V}_S = [V_1, V_2, \dots, V_M]^T$ to denote all voltage random variables in the network, where T denotes the transpose operator. At the discrete time n , the noiseless voltage measurement at bus i is $v_i[n] = |v_i[n]| \exp(j\theta_i[n]) \in \mathbb{C}$, where $|v_i[n]| \in \mathbb{R}$ denotes the voltage magnitude in per unit and $\theta_i[n] \in \mathbb{R}$ denotes the voltage phase angle in degrees. These measurements are in the steady state and all voltages are sinusoidal signals at the same frequency. We use $\mathbf{v}[n] = [v_1[n], v_2[n], \dots, v_M[n]]^T$ to denote a collection of all voltage measurements in a network at time n . Thus, $\mathbf{v}[n]$ is the realization of \mathbf{V}_S at time n . Also, we use $\mathbf{v}^{1:N} = (\mathbf{v}[1], \mathbf{v}[2], \dots, \mathbf{v}[N])$ to denote a collection of all voltage measurements in the network up to time N .

The problem to detect and localize line outages in a distribution grid is defined as follows:

- Problem: data-driven power outage detection and localization based on voltages
- Given: a sequence of N historical voltage measurements $\mathbf{v}^{1:N}$
- Find: (1) the outage time and (2) the branches that are out of service

III. SEQUENTIAL LINE OUTAGE DETECTION

A. Problem Formulation

Voltage measurements usually have an irregular distribution and are hard to be used for our goal of this paper. Therefore, instead of using voltage measurements directly, we use the incremental change of the voltage measurements to detect outages, which is defined as $\Delta\mathbf{v}[n] = \mathbf{v}[n] - \mathbf{v}[n-1]$. Accordingly, $\Delta\mathbf{v}^{1:N} = (\Delta\mathbf{v}[1], \Delta\mathbf{v}[2], \dots, \Delta\mathbf{v}[N])$. We use ΔV_i to represent the voltage change random variable at bus i and $\Delta\mathbf{V}_S$ to represent the voltage change random variables of the entire system. In the following, we will prove that, the probability distribution of $\Delta\mathbf{V}_S$ will be different after an outage. In the following context, the operator \setminus denotes the complement operator, i.e. $\mathcal{A} \setminus \mathcal{B} = \{i \in \mathcal{A}, i \notin \mathcal{B}\}$.

Theorem 1. *If the change of current injection at each bus is approximately independent and no branch connects bus i and bus j , the voltage changes at bus i and bus j are conditionally independent, given the voltage changes of all other buses, i.e. $\Delta V_i \perp \Delta V_j | \{\Delta V_e, e \in \mathcal{S} \setminus \{i, j\}\}$.*

Proof: For bus i , the current and voltage relationship can be expressed as $\Delta I_i = \Delta V_i Y_{ii} - \sum_{e \in \mathcal{N}(i)} \Delta V_e Y_{ie}$ with $Y_{ii} = \sum_{e \in \mathcal{N}(i)} Y_{ie}$, where Y_{ie} denote is the i eth element of the admittance matrix Y and the neighbor set $\mathcal{N}(i)$ contains the indices of the neighbors of bus i , i.e. $\mathcal{N}(i) = \{e \in \mathcal{S} | Y_{ie} \neq 0\}$. If bus i and bus j are not connected, $j \notin \mathcal{N}(i)$ and $Y_{ij} = 0$.

Given $\Delta V_e = \Delta v_e$ for all $e \in \mathcal{S} \setminus \{i, j\}$, the equation above becomes to

$$\begin{aligned}\Delta I_i &= \Delta V_i Y_{ii} - \sum_{e \in \mathcal{N}(i)} \Delta v_e Y_{ie}, \\ \Delta V_i &= \frac{1}{Y_{ii}} (\Delta I_i + \sum_{e \in \mathcal{N}(i)} \Delta v_e Y_{ie}).\end{aligned}$$

Similarly, $\Delta V_j = (\Delta I_j + \sum_{e \in \mathcal{N}(j)} \Delta v_e Y_{je}) / Y_{jj}$. With the assumption of the current change independence, i.e. $\Delta I_i \perp \Delta I_j$, ΔV_i and ΔV_j are conditionally independent given $\Delta \mathbf{V}_{\mathcal{S} \setminus \{i, j\}}$. ■

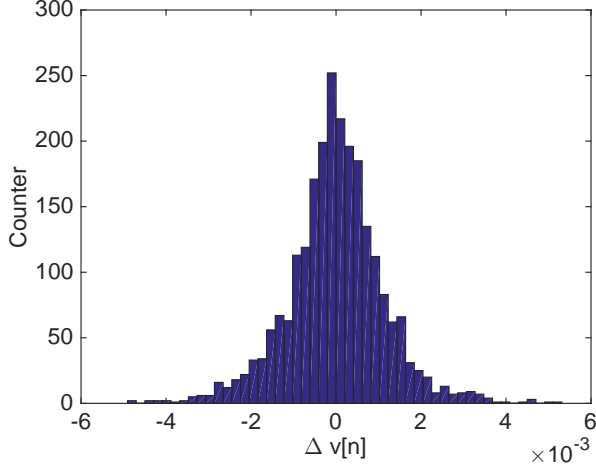


Fig. 1. Histogram of $\Delta v[n]$.

If a branch has an outage, its admittance becomes zero. The voltages at the two ends of this branch becomes conditionally independent. Assuming $\Delta \mathbf{V}_{\mathcal{S}}$ follows a multivariate Gaussian distribution, as empirically shown in Fig. 1, some elements of the mean vector and covariance matrix will change after an outage. Therefore, the probability distribution of $\Delta \mathbf{V}_{\mathcal{S}}$ is different before and after an outage. Let λ denote the time that an outage occurs. We assume that $\Delta \mathbf{V}_{\mathcal{S}}$ follow a Gaussian distribution g with the mean μ_0 and the covariance Σ_0 in the normal operation (i.e. $N \leq \lambda$) and the other Gaussian distribution f with the mean μ_1 and the covariance Σ_1 after any outage (i.e. $N > \lambda$). One way to find the outage time λ sequentially is to perform the following hypothesis test at each time N [14]:

$$\begin{aligned}\mathcal{H}_0 &: \lambda > N, \\ \mathcal{H}_1 &: \lambda \leq N.\end{aligned}$$

\mathcal{H}_0 represents that the outage has not occurred and \mathcal{H}_1 represents that the outage has occurred before N . Finding the outage time is known as the change point detection problem. If we assume the power outage time λ as a discrete random variable with a probability mass function $\pi(\lambda)$, we can use a Bayesian approach to find λ . In this paper, we assume λ follows a geometric distribution with a parameter ρ . The joint distribution of λ and $\Delta \mathbf{V}_{\mathcal{S}}$ can be written as

$$P(\lambda, \Delta \mathbf{V}_{\mathcal{S}}) = \pi(\lambda) P(\Delta \mathbf{V}_{\mathcal{S}} | \lambda).$$

When $\lambda = k$, all the data obtained before time k follow the distribution g and all the data obtained after time k

follow the distribution f . Therefore, the likelihood probability $P(\Delta \mathbf{V}_{\mathcal{S}} | \lambda)$ above is expressed as follows:

$$P(\Delta \mathbf{V}_{\mathcal{S}} = \Delta \mathbf{v}^{1:N} | \lambda = k) = \prod_{n=1}^{k-1} g(\Delta \mathbf{v}[n]) \prod_{n=k}^N f(\Delta \mathbf{v}[n]),$$

for $k = 1, 2, \dots, N+1$. When $\lambda = N+1$, it refers to the outage has not occurred.

In order to find the outage time λ , we can compare the pre-outage posterior probability $P(\mathcal{H}_0 | \Delta \mathbf{V}_{\mathcal{S}}) = P(\lambda > N | \Delta \mathbf{V}_{\mathcal{S}} = \Delta \mathbf{v}^{1:N})$ and post-outage posterior probability $P(\mathcal{H}_1 | \Delta \mathbf{V}_{\mathcal{S}}) = P(\lambda \leq N | \Delta \mathbf{V}_{\mathcal{S}} = \Delta \mathbf{v}^{1:N})$. One way to compare these posterior probabilities is using the posterior probability ratio at each time N , i.e.,

$$\begin{aligned}\Lambda(\Delta \mathbf{v}^{1:N}) &= \frac{P(\lambda \leq N | \Delta \mathbf{v}^{1:N})}{P(\lambda > N | \Delta \mathbf{v}^{1:N})} \\ &= \frac{\sum_{k=1}^N \pi(k) \prod_{n=1}^{k-1} g(\Delta \mathbf{v}[n]) \prod_{n=k}^N f(\Delta \mathbf{v}[n])}{\sum_{k=N+1}^{\infty} \pi(k) \prod_{n=k}^N g(\Delta \mathbf{v}[n])} \\ &= \frac{\sum_{k=1}^N \pi(k) \prod_{n=k}^N \frac{f(\Delta \mathbf{v}[n])}{g(\Delta \mathbf{v}[n])}}{\sum_{k=N+1}^{\infty} \pi(k)}.\end{aligned}\quad (1)$$

Before an outage occurs, the measurements are distributed with g , and $\Lambda(\Delta \mathbf{v}^{1:N})$ is small. After an outage, the measurements are distributed with f and $\Lambda(\Delta \mathbf{v}^{1:N})$ becomes large. Hence, we can set a threshold and declare an outage when the posterior probability ratio surpasses this threshold.

B. Optimal Outage Detection

In the change point detection problem, there are two performance metrics: *probability of false alarm* and *expected detection delay*. The former metric is the probability that a detector falsely declares an outage in the normal operation. If τ denotes the time of an outage being detected, the probability of false alarm is defined as $P(\tau < \lambda)$. The latter metric describes the average latency that a detector finds the outage after it has occurred. The expected detection delay is defined as $E(\tau - \lambda | \tau \geq \lambda)$. In power outage detection problem, we want to detect the outage happens at time λ as quickly as possible under a constraint of the maximum probability of false alarm. By the Shiryaev-Roberts-Pollaks procedure [23], for a maximum probability of false alarm α and the prior distribution parameter ρ , an optimal rule to detect outages is

$$\tau = \inf\{N \geq 1 : \Lambda(\Delta \mathbf{v}^{1:N}) \geq B_{\rho, \alpha}\}, \quad (3)$$

where $B_{\rho, \alpha} = (1 - \alpha) / \rho \alpha$ [24]. As shown in [14], this detection rule achieves an optimal expected detection delay asymptotically as α approaches zero. In many applications, the prior distribution parameter ρ is unknown. In these cases, we can assign an uninformative parameter, e.g. $\rho = 10^{-4}$. Now, the threshold is $B_{\rho, \alpha}$ is mainly decided by the false alarm rate α . As shown in Section V, this setup does not degrade the performance of the proposed algorithm. Lemma 1 shows the asymptotically optimal expected detection delay.

Lemma 1. For a given false alarm rate α , the detection rule in (3) achieves the asymptotically optimal detection delay

$$D(\tau) = E(\tau - \lambda | \tau \geq \lambda) = \frac{|\log \alpha|}{-\log(1 - \rho) + D_{KL}(f \| g)}, \quad (4)$$

as $\alpha \rightarrow 0$, where $D_{KL}(f||g)$ is the Kullback-Leibler distance [24].

In summary, when a new group of observation $\Delta \mathbf{v}[n]$ is available, we can compute the posterior probability ratio according to (2) and then apply the optimal detection rule in (3) to diagnose the system. Therefore, the proposed algorithm can be implemented for real-time outage detection.

C. Limitation of Optimal Outage Detection in Distribution Grids and Proposed Solution

To compute the posterior probability ratio in (2), we need to know both distributions g and f . Since the distribution g represents the voltage measurements in the normal operation, its parameters can be estimated based on the historical data that collected during the normal operation. However, the parameters of the post-outage distribution f are hard to obtain because any branch may have an outage during the normal operation. With the information on the distribution grid topology, experts may create a database to contain every possible line outage scenario and use the observed data to search the most likely distribution. However, this approach is infeasible for a large system. For a distribution grid with M buses and a radial topology, there are $M - 1$ branches. If one branch has an outage, there are $M - 1$ possible post-outage distributions. If two branches have outages, there are $(M - 1)(M - 2)$ possible post-outage distributions. Therefore, the number of possible distributions increases exponentially with the growth of possible line outages. Also, many DERs in the distribution grid do not belong to the utilities. Hence, their connectivities are unknown to the utilities, which makes it even harder to assume an accurate topology at the first step.

In this section, instead of searching the most likely post-outage distribution, we propose a method to learn f from data directly. The computational complexity of our approach is insensitive to the number of out-of-service branches. Specifically, we provide an approximation of the posterior probability ratio $\Lambda(\Delta \mathbf{v}^{1:N})$ in (2). With the Gaussian assumptions of both g and f , we can write the approximation of $\Lambda(\Delta \mathbf{v}^{1:N})$ in a closed form. Then, we use the maximum likelihood method to estimate the mean and covariance of f . Since the outage time λ is unknown, we use all the measurements $\Delta \mathbf{v}^{1:N}$ to estimate the distribution parameters. Once the parameters of f are estimated, we compute $\Lambda(\Delta \mathbf{v}^{1:N})$ with the estimated parameters and use the optimal detection rule in (3) to find outages. As shown in Section V, using these estimated parameters to detect outages can also achieves the optimal expected detection delay.

The posterior likelihood ratio $\Lambda(\Delta \mathbf{v}^{1:N})$ in (2) can be expressed as

$$\log(\Lambda(\Delta \mathbf{v}^{1:N})) = \log \left(\frac{\sum_{k=0}^N \pi(k) \prod_{n=k}^N \frac{f(\Delta \mathbf{v}[n])}{g(\Delta \mathbf{v}[n])}}{\sum_{k=0}^N \pi(k)} \right) + C, \quad (5)$$

where $C = \log \left(\frac{\sum_{k=0}^N \pi(k)}{\sum_{k=N+1}^{\infty} \pi(k)} \right)$ and $\pi(k) = P(\lambda = k)$ is the distribution of the outage time variable λ . In (5), $\sum_{k=0}^N \pi(k) \prod_{n=k}^N \frac{f(\Delta \mathbf{v}[n])}{g(\Delta \mathbf{v}[n])}$ can be regarded as the expectation of $\prod_{n=k}^N \frac{f(\Delta \mathbf{v}[n])}{g(\Delta \mathbf{v}[n])}$ over the prior distribution π , i.e.,

$E_{\pi} \left(\prod_{n=k}^N \frac{f(\Delta \mathbf{v}[n])}{g(\Delta \mathbf{v}[n])} \right)$. Also, the logarithmic function is convex. Hence, we can apply the Jensen's inequality to approximate $\log(\Lambda(\Delta \mathbf{v}^{1:N}))$, i.e.,

$$\begin{aligned} & \log(\Lambda(\Delta \mathbf{v}^{1:N})) \\ & \geq \frac{\sum_{k=0}^N \pi(k) \sum_{n=k}^N (\log(f(\Delta \mathbf{v}[n])/g(\Delta \mathbf{v}[n])))}{\sum_{k=0}^N \pi(k)} + C \\ & \triangleq \tilde{\Lambda}(\Delta \mathbf{v}^{1:N}). \end{aligned} \quad (6)$$

If we assume g and f as Gaussian distributions with the parameters defined previously, we can further simplify $\tilde{\Lambda}(\Delta \mathbf{v}^{1:N})$, which becomes

$$\begin{aligned} \tilde{\Lambda}(\Delta \mathbf{v}^{1:N}) &= \frac{1}{\sum_{k=0}^N \pi(k)} \left\{ \sum_{k=0}^N \frac{\pi(k)}{2} \sum_{n=k}^N \log |\Sigma_1| \right. \\ & \quad + \log |\Sigma_0| + (\Delta \mathbf{v}[n] - \mu_1)^T \Sigma_1^{-1} (\Delta \mathbf{v}[n] - \mu_1) \\ & \quad \left. - (\Delta \mathbf{v}[n] - \mu_0)^T \Sigma_0^{-1} (\Delta \mathbf{v}[n] - \mu_0) \right\} + C. \end{aligned} \quad (7)$$

Now, with the simplification in (7), we can use the maximum likelihood method to estimate both μ_1 and Σ_1 by setting $\partial \tilde{\Lambda}(\Delta \mathbf{v}^{1:N}) / \partial \mu_1 = 0$ and $\partial \tilde{\Lambda}(\Delta \mathbf{v}^{1:N}) / \partial \Sigma_1 = 0$. Specifically, to estimate μ_1 , we have

$$\begin{aligned} \frac{\partial \tilde{\Lambda}(\Delta \mathbf{v}^{1:N})}{\partial \mu_1} &= \frac{\sum_{k=0}^N \pi(k) \sum_{n=k}^N (\Delta \mathbf{v}[n] - \mu_1) \Sigma_1^{-1}}{\sum_{k=0}^N \pi(k)} \\ &= \frac{\sum_{k=0}^N \pi(k) \sum_{n=k}^N \Delta \mathbf{v}[n] - (N - k + 1) \mu_1}{\Sigma_1 \sum_{k=0}^N \pi(k)}. \end{aligned}$$

Therefore, the estimate of the mean vector μ_1 is

$$\hat{\mu}_1 = \frac{\sum_{k=0}^N \pi(k) \sum_{n=k}^N \Delta \mathbf{v}[n]}{\sum_{k=0}^N \pi(k) (N - k + 1)}. \quad (8)$$

Similarly, the covariance matrix Σ_1 can be estimated as

$$\begin{aligned} \frac{\partial \tilde{\Lambda}(\Delta \mathbf{v}^{1:N})}{\partial \Sigma_1} &= \frac{-1}{2 \sum_{k=0}^N \pi(k)} \\ & \quad \cdot \sum_{k=0}^N \pi(k) [(N - k + 1) \text{tr}(\Sigma_1^{-1} (\partial \Sigma_1)) \\ & \quad - \text{tr}(\Sigma_1^{-1} (\partial \Sigma_1)^{-1} \Sigma_1^{-1} S_k)] = 0, \end{aligned}$$

where $S_k = \sum_{n=k}^N (\Delta \mathbf{v}[n] - \mu_1)(\Delta \mathbf{v}[n] - \mu_1)^T$. If we let $\mu_1 = \hat{\mu}_1$, the covariance matrix estimate is

$$\hat{\Sigma}_1 = \frac{\sum_{k=0}^N \pi(k) S_k}{\sum_{k=0}^N \pi(k) (N - k + 1)}. \quad (9)$$

Although we assume that both g and f follow Gaussian distributions, the maximum likelihood parameter estimation method presented above can be generalized for a boarder range of distributions, such as exponential family distributions. This extension will help us to exam more complicated scenarios in power systems. Once $\hat{\mu}_1$ and $\hat{\Sigma}_1$ are estimated, we can use them to compute the posterior probability ratio $\Lambda(\Delta \mathbf{v}^{1:N})$ and then apply the optimal detection rule in (3).

IV. LINE OUTAGE IDENTIFICATION

Identifying which line has an outage is important in the urban distribution grid operation. In metropolitan areas, many branches are underground and not well documented. For example, in New York City, the total length of underground power cables is about 94000 miles [12]. Therefore, an efficient and accurate outage localization approach can reduce the power interruption time significantly. In the following part, we will propose a real-time outage localization method based on the voltage measurements.

Because of Theorem 1, the voltage changes at the two ends of the out-of-service branches are conditionally independent after an outage. With the assumption of Gaussian distribution, the conditional covariance of the voltages will be zero after an outage. Therefore, we can compute the conditional covariance matrix of every possible pair of buses in the network and check if the off-diagonal term changes to zero. When the off-diagonal term changes to zero, we can identify the out-of-service branches.

Usually, the conditional covariance can be estimated based on the voltage measurements. However, a large set of post-outage data is required to have an accurate estimation, and the delay of localization is long. To enable real-time outage localization, we use the covariance matrix Σ to compute the conditional covariance alternatively. This approach allows us to localize the outage even if we do not know the distribution grid topology. In the case that the post-outage probability distribution f is unknown, we can use $\widehat{\Sigma}_1$ in (9) to compute the conditional covariance. For bus i and bus j , suppose $\mathcal{I} = \{i, j\}$ and $\mathcal{J} = \mathcal{S} \setminus \{i, j\}$, the covariance of the joint Gaussian distribution can be decomposed as

$$\Sigma = \begin{bmatrix} \Sigma_{\mathcal{I}\mathcal{I}} & \Sigma_{\mathcal{I}\mathcal{J}} \\ \Sigma_{\mathcal{I}\mathcal{J}}^T & \Sigma_{\mathcal{J}\mathcal{J}} \end{bmatrix}.$$

The conditional covariance matrix can be computed by the Schur complement [25], i.e.,

$$\Sigma_{\mathcal{I}|\mathcal{J}} = \Sigma_{\mathcal{I}\mathcal{I}} - \Sigma_{\mathcal{I}\mathcal{J}}\Sigma_{\mathcal{J}\mathcal{J}}^{-1}\Sigma_{\mathcal{I}\mathcal{J}}^T. \quad (10)$$

If the voltages at bus i and bus j are conditionally independent, the off-diagonal term of $\Sigma_{\mathcal{I}|\mathcal{J}}$ is zero, i.e. $\Sigma_{\mathcal{I}|\mathcal{J}}(1,2) = \Sigma_{\mathcal{I}|\mathcal{J}}(2,1) = 0$. Therefore, we can compare the conditional covariance of every bus pairs before and after an outage. If the conditional covariance changes to zero after an outage, we localize one line outage. This computation can be repeated when $\widehat{\Sigma}_1$ is updated based on the latest available measurements. In Section V, we illustrate the similar performances using the true post-outage covariance matrix Σ_1 and the estimated covariance matrix $\widehat{\Sigma}_1$.

Let the estimated out-of-service branch set be denoted as $\widehat{\mathcal{E}}_{\text{outage}}$. We summarize the proposed outage detection and localization algorithm in Fig. 2.

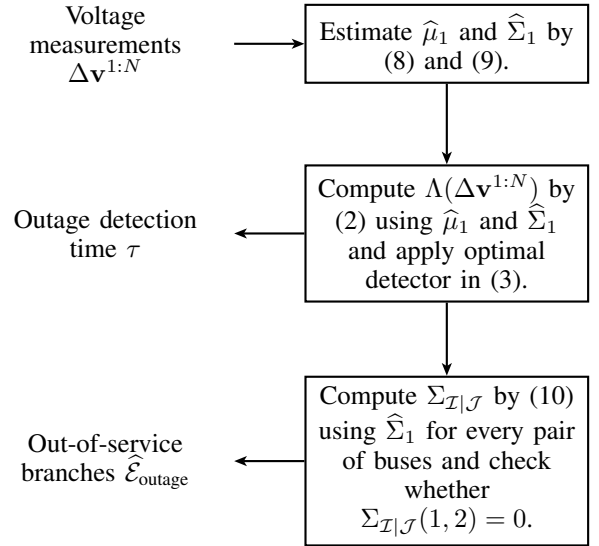


Fig. 2. Flow chart of the proposed outage detection and localization algorithm.

V. SIMULATION AND RESULTS

The simulations are implemented on the IEEE PES distribution networks for IEEE 8-bus and 123-bus networks [22]. To validate the performance of the proposed approach on loopy networks, we add several branches to create loops in both systems. The loopy 8-bus system is shown in Fig. 3. For 123-bus system, we add a branch between bus 77 and bus 120 and the other branch between bus 50 and bus 56. The admittances are as same as the branch between bus 122 and bus 123. In each network, bus 1 is selected as the slack bus. The historical data have been preprocessed by the MATLAB Power System Simulation Package (MATPOWER) [26]. To simulate the power system behavior in a more realistic pattern, we use the real power profile from Pacific Gas and Electric Company (PG&E) in the subsequent simulation. This profile contains anonymized and secure hourly smart meter readings over 110,000 PG&E residential customers for one year spanning from 2011 to 2012. The reactive power $q_i[t]$ at bus i and time t is computed according to a randomly generated power factor $pf_i[t]$, which follows a uniform distribution, e.g. $pf_i[t] \sim \text{Unif}(0.9, 1)$. The results presented in the previous sections are based on the complex voltage. However, the voltage angles are hard to obtain due to the limited deployment of micro-PMU. In this simulation, we only consider voltage magnitude. As shown in the following parts, using voltage magnitude can also achieve the expected optimal detection delay. To obtain voltage magnitude at time t , i.e. $|v_i[t]|$, we run a power flow to generate the states of the power system. To obtain time-series data, we run the power flow to generate hourly data over a year.

In this simulation, we considered three common outage scenarios:

- A loopy network. In this system, after an outage, most buses will not have zero voltages because they can receive powers from multiple branches. This outage scenario usually happens in the urban areas.

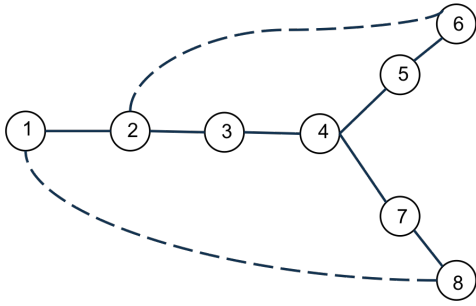


Fig. 3. 8-bus system. Node represents bus and line represents branch. The dashed lines are additional branches with the same admittance as the branch connected bus 7 and bus 8.

- A radial network with DERs. In this case, some buses will be disconnected from the main grid. However, if they are connected with DERs, such as solar panels and batteries, their voltages will not be zero. This outage case is a typical scenario in the residential areas.
- A radial network without DERs. In this case, when a line outage occurs, some buses will be disconnected from the main grid and have zero voltage magnitudes. Because the bus voltages have no variation after outages, our method can quickly detect and localize this type of outages.

A. Outage Detection in Loopy Systems

Fig. 4 illustrates the posterior probability ratio $\Lambda(\Delta \mathbf{v}^{1:N})$ for detecting two line outages in loopy 8-bus system (Fig. 3). In this test, Branches 3-4 and 2-6 have outages. The false alarm rate α is 10^{-5} . To have a better understand of how our proposed outage detection algorithm works, we assign an uninformative parameter for the prior distribution, i.e. $\rho = 10^{-4}$. Therefore, the threshold $B_{\rho, \alpha} \simeq 10^{11}$. The outage time is $\lambda = 21$. We can observe that both posterior probability ratios, with and without knowing the post-outage distribution f , immediately across the threshold $B_{\rho, \alpha}$ at $N = 21$. Therefore, both methods have zero detection delay. In addition, Fig. 4 illustrates that using the estimated parameters to detect outages performs identically as the optimal method with known f .

In Fig. 5 and Fig. 6, the expected delay divided by $|\log \alpha|$ is plotted as a function of $|\log \alpha|$ for two cases: f is known and f is unknown. We also show the limiting value of the normalized asymptotically optimal detection delay as predicted by Lemma 1. All plots are generated by Monte Carlo simulation over 1000 replications. In this simulation, the prior is geometric with parameter $\rho = 0.04$. The start time of test is randomly selected within one year. In Fig. 5, our approach, which learns the parameters of the post-outage distribution from the voltage measurements, has identical performances as the optimal method that has known f . Also, our approach can achieve the optimal expected detection delay asymptotically. As shown in Fig. 5, when the false alarm rate α is small, our approach can report the outage immediately (i.e. detection delay is less than one), which can significantly reduce the impacts of power outages. Similarly results can be obtained from Fig. 6, where a loopy 123-bus system is simulated.

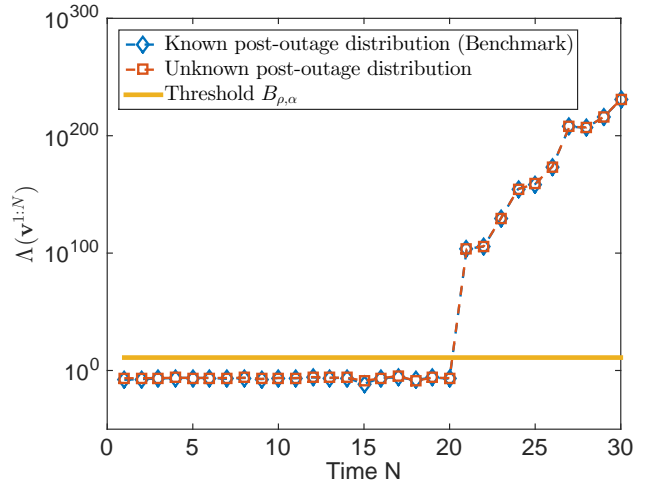


Fig. 4. Test statistics for the sequential probability ratio tests. The branches 3-4 and 2-6 have outage. $\alpha = 10^{-5}$, $\rho = 10^{-4}$.

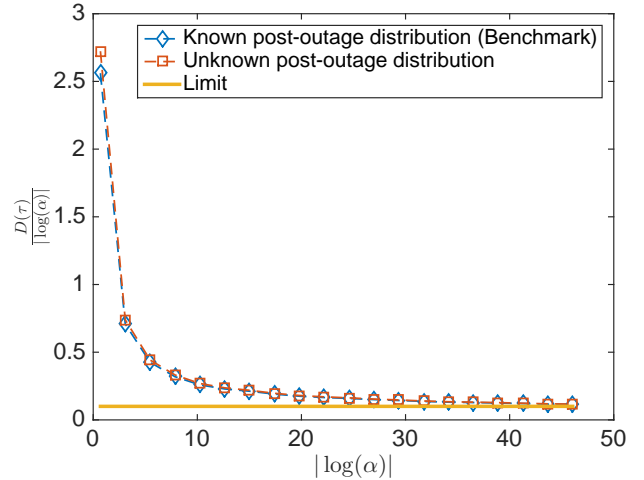


Fig. 5. Plots of the slope $\frac{1}{|\log \alpha|} E[\tau - \lambda | \tau \geq \lambda]$ against $|\log \alpha|$ for outage detection for loopy 8-bus system. False alarm rate α ranges in $[0.5, 10^{-20}]$. Branch 3-4 has an outage.

B. Radial Network with DERs

In a radial distribution grid, a line outage will lead to several isolated islands. However, with the integration of DERs, such as solar panels and batteries, some buses can still receive powers. In this simulation, we assume that for the radial 8-bus system in Fig. 3, branch 3-4 has an outage, and there are two islands. Also, we assume bus 8 is a solar power generator with a battery as the storage. Thus, there is a power supply during the entire day. If the battery is not available, the outage can be directly detected when the nodal voltages are zero. For the solar panel, we use the hourly power generation profile computed by PVWatts Calculator, an online application developed by the National Renewable Energy Laboratory (NREL) [27]. The solar power generation profile are computed based on the weather history in North California and the physical parameters of ten 5kW solar panels.

In Fig. 7, the expected delay divided by $|\log \alpha|$ is plotted

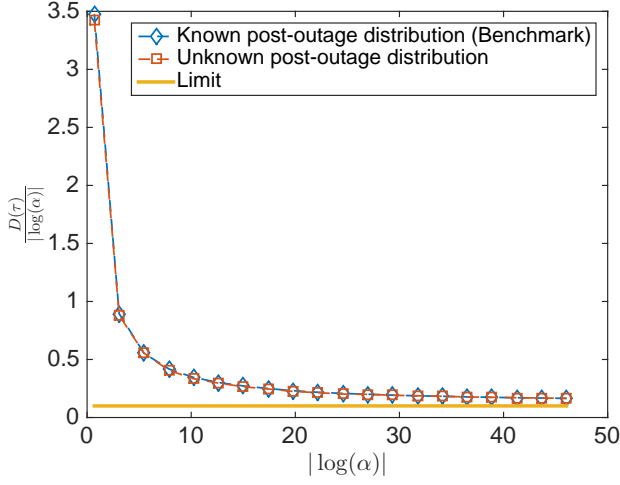


Fig. 6. Plots of the slope $\frac{1}{|\log \alpha|} E[\tau - \lambda | \tau \geq \lambda]$ against $|\log \alpha|$ for outage detection for loopy 123-bus system. False alarm rate α ranges in $[0.5, 10^{-20}]$. Branch 73-74 has an outage.

as a function of $|\log \alpha|$. We can observe that the proposed method, which does not require the post-outage distribution f , has identical performance as the detectors with known f . This result highlights the outage detection capability of the proposed algorithm in the present of DERs.

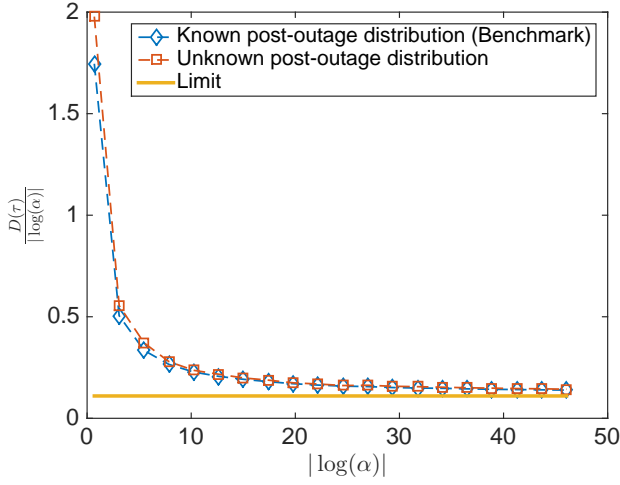


Fig. 7. Plots of the slope $\frac{1}{|\log \alpha|} E[\tau - \lambda | \tau \geq \lambda]$ against $|\log \alpha|$ for outage detection for 8-bus system without loop. False alarm rate α ranges in $[0.5, 10^{-20}]$. Branch 3-4 has an outage.

C. Outage Identification

Since the variation of voltage magnitudes is small, the conditional covariance is hard to visualize. Alternatively, we show the absolute conditional correlation in this section. For bus i and bus j , their conditional covariance matrix is denoted as $\Sigma_{\mathcal{I}|\mathcal{J}}$, where $\mathcal{I} = \{i, j\}$ and $\mathcal{J} = \mathcal{S} \setminus \{i, j\}$. The conditional correlation between bus i and bus j is defined as

$$\rho_{i,j} = \frac{\Sigma_{\mathcal{I}|\mathcal{J}}(1,2)}{\sqrt{\Sigma_{\mathcal{I}|\mathcal{J}}(1,1) \times \Sigma_{\mathcal{I}|\mathcal{J}}(2,2)}}.$$

When a branch has an outage, the conditional correlation becomes zero. Fig. 9 shows the absolute conditional correlation $|\rho_{i,j}|$ of the loopy 8-bus system in Fig. 3 after branch 3-4 and branch 2-6 have outages. The red boxes indicate the branches that have outages. We assume that the post-outage distribution f is known, which is an ideal and optimal approach. Thus, the true Σ_1 is used to compute the conditional correlation. Compared with Fig. 8, clearly, the absolute conditional correlations of outage branches change to zero after outages. The diagonal terms are the self-correlation and equal to one. This observation indicates that this proposed outage localization method is sensitive to outages. Also, it validates our proof in Theorem 1.

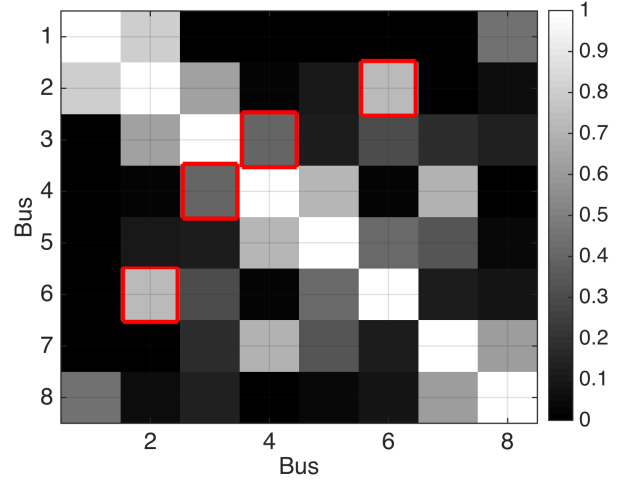


Fig. 8. Absolute conditional correlation of 8-bus system before an outage.

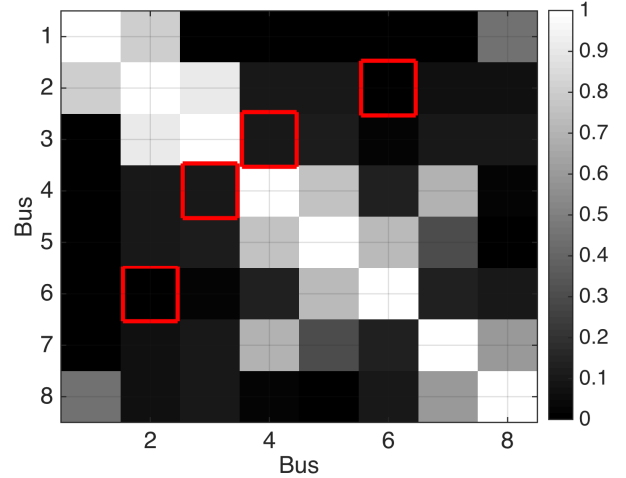


Fig. 9. Absolute conditional correlation of 8-bus system after an outage. Branches 3-4 and 2-6 have outages. f is known.

Fig. 10 shows the post-outage conditional correlation of the same system when f is unknown, which is a realistic case. The estimated covariance matrix $\hat{\Sigma}_1$ is used to compute the conditional correlation. We can observe that the outage branches have a significant change of the conditional correlation. Also,

compared with Fig. 9, most terms of the conditional correlation are similar. These results show that although f is unknown, the proposed method can still localize the out-of-service branches as accurate as the optimal approach.

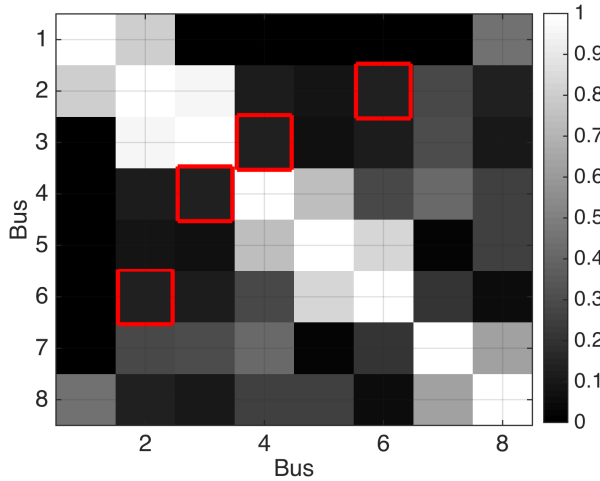


Fig. 10. Absolute conditional correlation of 8-bus system after an outage. Branches 3-4 and 2-6 have outages. f is unknown.

VI. CONCLUSION

In this paper, we propose a data-driven algorithm to automatically detect and identify outages in distribution grids with increasing renewable penetration. Specifically, we develop a stochastic modeling of smart meter data stream and propose a change point detection approach based on the probability distribution changes because of outage events. As a highlight, unlike existing approaches, our method neither requires a tree structure nor a passive distribution grid. This leads to wide applicability of our proposed method than existing methods. In addition to outage detection, we provide theoretical prove that optimal identification can be achieved due to the conditional independence of voltages based on the power flow analysis. We verify the proposed algorithm on both IEEE 8- and 123-bus systems with and without DERs. From extensive simulations, our algorithm can perfectly detect and identify outages in a short time, with and without the integration of DERs.

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